School of Science and Engineering

RISK MANAGEMENT AND PORTFOLIO OPTIMIZATION USING R

Final Capstone Report

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By

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Student Statement:

I, Leila Ouellaj, certify that this project reflects my own research and work. I certify that I applied the ethics of design, and rules of safety of the public, and none of them was violated.

Leila Ouellaj

Approved by the Supervisor

Dr. Lahcen Laayouni
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ABSTRACT

This paper represents my capstone report about “Risk Management and Portfolio Optimization using R”. The financial sector in Morocco can be characterized as illiquid due to many reasons although the government is making a lot of effort to change the latter. The model presented in this report is an attempt to encourage domestic investment. The model uses the Markowitz portfolio theory that puts the variance as the main equation of risk along with some constraints, and solves it using the Lagrange multipliers. Based on the historical data of 2018, we will create different portfolio combinations for investors to choose from depending on the required return, and the level of risk they’re willing to take. We will try to optimize different types of portfolios through getting the best combination of weights to be allocated to each asset by coding the mathematical model using R language through RStudio. This approach can minimize the risk greatly and push investors to buy more stocks. However, there is a type of risk that is inevitable and that cannot be diversified, which is the systematic risk. Real life simulations through an online investment platform will be used to verify our results. Finally, this paper also tackles the STEEPLE implications of the model developed in order to explain the added value of the project.

Keyword: simulation, portfolio optimization, investment, STEEPLE, Lagrange multipliers, Morocco.
RESUME

Ce document représente mon rapport de synthèse sur «La gestion des risques et l’optimisation de portefeuille avec R». Le secteur financier marocain peut être qualifié d’illiquide pour de nombreuses raisons, bien que le gouvernement déploie de nombreux efforts pour changer ce dernier. Le modèle présenté dans ce rapport est une tentative d’encourager les investissements nationaux. Le modèle a utilisé la théorie du portefeuille de Markowitz, qui place la variance comme principale équation du risque ainsi que certaines contraintes, et la résout à l’aide des multiplicateurs de Lagrange. Sur la base des données historiques de 2018, nous créerons différentes combinaisons de portefeuille parmi lesquelles les investisseurs pourront choisir en fonction du rendement souhaité et du niveau de risque qu’ils sont prêts à assumer. Nous allons essayer d’optimiser différents types de portefeuilles en obtenant la meilleure combinaison de pondérations à attribuer à chaque actif en codant le modèle mathématique à l’aide du langage R via RStudio. Cette approche peut considérablement réduire les risques et inciter les investisseurs à acheter plus d’actions. Cependant, il existe un type de risque inévitable qui ne peut être diversifié, à savoir le risque systématique. Des simulations de la vie réelle via une plateforme d’investissement en ligne seront utilisées pour vérifier nos résultats. Enfin, cet article aborde également les implications STEEPLE du modèle développé afin d’expliquer la valeur ajoutée du projet.

Mot-clés: Maroc, optimisation, simulation, investissement, portefeuille, Lagrange, STEEPLE
Introduction
In the last years, the issue of modeling financial risk became very important in the financial domain. Furthermore, the interest in forecasting investment risk has increased significantly as well. This is mainly due to the change in laws and regulations of the financial and banking sector. As a result, many researchers such as Markowitz worked on developing mathematical models that would best respond to investors’ questions concerning risk and assets allocation. One should keep in mind that the models rely on estimated variables, that can create errors. These errors should be taken into account when making an investment decision.

1. Project Description
The objective of this capstone design project is to use mathematical method to model the risk and methods to optimize a portfolio. The goal would be to maximize the profit of the portfolio and minimize its risk. In order to complete this task, the data used will be directly coming from the Casablanca Stock Exchange Website, as well as companies’ individual yearly public reports. The volatility and risk of return of companies’ stocks will be taken into account when optimizing the portfolio risk. The first step would be to focus on three publically listed Moroccan corporations. Because the final objective is to find the appropriate weights to assign to each investment taking into consideration its volatility, the mathematical model to be used will be the Lagrange Methods. The data will be downloaded from Casablanca Stock Exchange website, then it will be uploaded to RStudio. The scenarios will be generated using Lagrange Equations. Using the resultant weights, the interpretation of the results should be done in common language. The next step would be to use R to code the entire risk management and portfolio optimization process. The process would include all the Lagrange Equations and the user will only be asked to provide the rate of return and volatility of investments. The program will then generate the optimal investment.
The main objective of this capstone is to develop a simple, easy and ready to use algorithm by the Moroccan investors to help them make an informed decision about their investments. By doing so, we will be encouraging domestic investment, increasing the liquidity in the Moroccan market and developing the overall economy.

2. Investment in the Moroccan Context

In the Moroccan context, the amount invested and reinvested have been historically going up as shown below since 2010.

![Figure 1: Amounts Invested in Morocco](chart)

The drop in investments in 2017 is partly attributed to the postponement of several transactions to 2018 to benefit from the incentive on the registration fee. Professionals anticipate a rebound in the amounts invested. The start of the seed funds will be felt more on the number of transactions. However, it seems that investors don’t trust the system of investment and don’t follow a model for investment. It is rather random where they rely on historical data such as rate of return. Many financial analysts explain that the current investment model in Morocco is failing because there is not an actual model. Another issue is that investors prefer to head towards financial products of big companies only.
The latest data shows that the stock exchange market in Morocco moved from 583 billion of Dirhams in 2016 to 626 billion of dirhams in 2017, meaning an increase of 7.54%. Currently, most of the stocks exchanged are the banking sectors followed by the telecommunication one as shown in the figure below.

Concerning the liquidity in the stock market, it’s currently at 10.38% which means that the volume of stocks exchanged is relatively low.

The AMMC (Autorité Marocaine des Marchés des Capitaux) regulates the investments in Morocco, its website gives general guidelines for investment in general not specific to the Moroccan market. The only important point is diversification, however they don’t give specific guidelines as how to diversify or mathematical model to use. It presents regulations in place for internal and external risk reduction. There is not a mathematical accurate model for risk management. They rely on historical reports and risks to manage future risks.
3. Financial Products in Morocco

In order to invest in Morocco, most people go through brokerage firms. On the AMMC website, 17 companies are listed, however less than 10 cater to retail clients.

Logging into the CSE website (Casablanca Stock Exchange), two main types of financial products are present; stocks and bonds. Options are also available however no brokerage firm proposes this product. Stocks are the most traded financial product in Morocco. Currently, the 10 most active shares are: [1]

- Itissalat Al-Maghrib with a volume of 38 205 854.9.
- BCP with a volume of 37 316 632.65.
- ATTIJARIWAFABANK with a volume of 22 134 038.
- LAFARGEHOLCIM MAR with a volume of 5 947 375.
- Societe Des Boissons Du Maroc with a volume of 4 819 725.
- Mutandis SCA with a volume of 4 410 617.05.
- BMCE Bank with a volume of 3 424 267.5.
- COSUMAR with a volume of 3 165 359.75.
- SODEP-Marsa Maroc with a volume of 2 907 372.75.

Recently, the AMMC has been working on issuing new types of bonds which are Green, Social & Sustainability Bonds.

- **Green bonds** are bonds whose product is exclusively earmarked for the financing or refinancing, in whole or in part, of new projects or existing projects with positive environmental impacts. There is a wide range of activities and projects with a positive environmental impact and can therefore be eligible for Green Bond financing. However, international standards establish, with different degrees of detail, lists or taxonomies of eligible projects. As an example, ICMA sets, in its Green Bonds Principles, a list of...
general categories of eligible projects while the CBI proposes a taxonomy more detailed to qualify for CBI certification. [2]

- **Social bonds**, are any obligation the proceeds of which are exclusively for the financing or refinancing of all or part of new projects existing projects aimed directly at solving or mitigating social problems specific and / or targeting a positive social impact, specifically but not exclusively benefit of one or more target population. Social projects eligible for financing by a social obligation must have clear social benefits that can be assessed, and quantified if possible by issuers. Thus, the notion of the target population becomes decisive in the context of Social Bonds. In Indeed, the issuer of a Social Bond must imperatively specify the target population beneficiaries of the social impacts of the projects concerned by the program. [2]

- **Sustainability bonds**, are bonds whose issuance proceeds is intended to finance or refinance a combination of green projects (with positive environmental impact) and social (positive social impact for a target population). So, their definition is aligned with Green Bonds and Social Bonds. Indeed, since the sustainability approach is often an integrated approach, some issuers could raise debt in the market to finance a combination of green and social. In addition, some green projects may have social benefits and vice versa, some social projects can also have positive environmental impacts. [2]

### 4. Portfolio and Risk Management

To define a portfolio, it is a set of securities that need to be diversified, and which individual weights combine to a sum of 1. Weights may be positive to represent a long position or negative to represent a short position. The goal is to provide diversification and reduce risks because one cannot know which stock is simply the best, and which one will provide higher rate of return. The most important characteristic about a portfolio is risk and reward, investors seek high
reward and lower risk. The rate of return is what is meant by reward, higher rewards are always the ultimate goal. While the risk is the probability of default from the company resulting in a negative return. Volatility is also a concept that investors dislike, high volatility a lot of times means more risk and uncertainty about the rate of return. When investing in a diversified portfolio, only the overall return and volatility of the portfolio can be expected, not individual stocks. Investing in an optimal portfolio starts by choosing the weight of each stock. The overall variance of the portfolio should be low while the expected return should be high. The main method to calculate a portfolio return and variance is by using the mean-variance analysis through the Lagrange Equations.

5. Volatility in the Moroccan Stock Market
There was no reasonable example of basic increment in volatility on the Moroccan market, aside from the presence of noteworthy momentary increases. It is the outcome, more than the reason, of the unsteadiness of the Moroccan financial market. The development of arbitrage between investment instruments increases overall liquidity thus reducing volatility. The Moroccan stocks market is known for its illiquidity as well as low volatility. However, volatility should not be considered as a good sign in this case, it can be highly biased by the illiquidity of the market. [3] The market volatility can also be influenced by the investor’s behavior as to the diversification of the assets. Then, the unchanged price of assets is mainly a consequence of the market being illiquid, not to a fluctuation in the company’s activities or shares.

Bank Al-Maghrib (BAM), the Moroccan Capital Market Authority (AMMC) and the Insurance and Social Welfare Control Authority (ACAPS) have released the latest report on financial stability for the financial year 2017. This report sets out, among other things, the state of the capital market. This is stable in its entirety, despite the decline in the volume of capital raising in 2017, the low liquidity of the Casablanca market and its price compared to other places. In detail, the raising of capital on the stock market is becoming weaker.
The primary market was not very active with a volume of capital raised of 1.4 billion dirhams, in the form of capital increases, against 2 billion in 2016 and 7.8 billion in 2011. [4]

On the other side, the secondary market remains marked by a deficit of liquidity mainly due to a low free float, despite the increase of 31% of securities traded on the stock market which rose from 217 million in 2016 to 284 million shares traded in 2017. [4]

In more detail, the liquidity ratio remains low, despite a slight upward trend over the last three years. It stood at 10.4% for the year 2017, against 9.5% a year earlier. [4]

Note that the liquidity ratio is calculated on the basis of the stock turnover rate in relation to the total market capitalization.

This situation is explained in particular by the weakness of the floating factor on the stock market. The preponderance of strategic shares held by stable shareholders limits the number of shares that are likely to be traded on the stock market in the short term.

The weakness of the liquidity of the Casablanca Stock Exchange jumps to the eye even more when compared to other stock markets. The liquidity of the Casablanca rating is remarkably lower than that of South Africa and Egypt, which have a liquidity ratio of 38.4% and 39.2% respectively. The gap becomes larger when compared with other emerging markets such as Brazil (73.6%) and Turkey (168.6%). [4]

A large gap exists between supply and demand prices. Regarding the "securities offered / securities demanded" ratio, the ratio was 115.1%, which means that the number of securities offered for sale is greater than the number of securities requested at the time of purchase.

For their part, the supply and demand satisfaction rates, which remain limited, improved slightly in 2017, which are respectively 36.7% and 42.2%, compared with 32.5% and 27.7% in 2016. This situation reveals that the gap between the prices of supply and those of demand remains important. [4]
The volatility of the MASI and MADEX indices is 10.66% and 11.22% in 2017, compared with 9.84% and 10.20% in 2016. Nevertheless, compared to other Casablanca locations, level of volatility of the Casablanca place remains moderate. [4]

The Price Earnings Ratio of the Casablanca Stock Exchange is high. The valuation of securities listed on the Casablanca Stock Exchange remains high overall with a PER at 19.7x, although a slight decrease compared to 2016 (20.3x).

This decrease is explained by a profit share of listed companies (+11%) which increased faster than the overall valuation (+8%). Thus, 12% of listed securities have a significant discount compared to the market PER, and 4% of the values show a significant premium. [4]

The high price of Casablanca's place can be explained by the still low interest rate environment where investors are looking for financial instruments with a better return. This situation increases the risk of instability in the stock market, especially if the context of low interest rates were to change drastically.

6. Optimal Investment Portfolios: Portfolio theory

The trade of securities has been rising steeply in emerging economies’ markets. Although these markets have some resemblances in the path to development, the differences are what usually attracts investors because they present special expected returns and risks that cannot be found in others markets, these risks can be managed through risk management tools. Managing risk is an essential part of the financial world, success cannot happen without it. Of course, it has to be within a legal frame and accounting principled need to be respected. In parallel, all investment players will be able to make more profit and decrease the risk of default.

During the years of the 1950s, Markowitz put into life the modern portfolio theory where he described the efficient portfolio. Markowitz theory is about the weights meaning the amount of money from the investors wallet that will be reserved for each asset. Markowitz’s student
named Sharpe, in 1963, worked on developing a single index model that combines return on common stocks such as DJI. Moreover, more developed risk management techniques such as mean-variance and value at risk (VAR) are actually not very recent and are all inspired from the modern portfolio theory first described by Markowitz. Today, the idea of the VAR is very popular and used widespread thanks to J.P Morgan. Many applications of risk management and common funds use the portfolio theory as well as the VAR concept. [5]

The requests for risk tools and procedures by emerging nations are currently huge and it will keep on developing all through the 21st century. This will put a larger weight on banking officers and to guarantee the security and soundness of their particular finance related frameworks through compelling guidelines. This, obviously, is a troublesome assignment, this has been confirmed throughout the last financial crises. Looking forward, the errand is probably going to get more difficult, given the possible increment in the intricacy of financial instruments and the greatness of cross outskirt financial streams. Starting and emending consistence or normal standards of the finance game is a challenging try for any controller in the money market. [5]

Exchanging comprises of exclusive positions in financial instruments which are showcased for repurchase and which are taken on by the budgetary substance with the goal of profiting by genuine and anticipated contrasts among buy and deal costs or from other price fluctuations, for example, spread differentials. Exchanging risk is characterized as the risk that the exchanging income will diminish because of an unfavorable value chance in the exchanged asset. [5]

After several consecutives financial losses in the world, approaching systematically trade risk management has become very important. Investors are looking for a well-established system of reporting to evaluate the risk. For the latter reason, investments banks and brokerage firms have relied heavily on the Portfolio Theory. However, it’s still in the development phase in Morocco.
7. Methodology

7.1 Portfolio Theory solved using Lagrange

A portfolio having \( n \) assets where \( w_i \) represents the weight allocated to each asset, then:

\[
\sum_{i=1}^{n} w_i = 1
\]

As mentioned in the previous part, the weights can be negative assuming short positions.

The goal is to determine the weight to be allocated for each asset in order to minimize the risk.

The rate of return of the portfolio can be expressed as such, where \( R_i \) is the rate of return of an asset \( i \), then \( \mu_i = R \) and \( \sigma_i^2 = \text{var}(R_i) \) are the expected value and variance of \( R_i \) for \( i=1:n \). Let \( \rho_{i,j} \) be the correlation between the rates of return:

\[
R = \sum_{i=1}^{n} w_i R_i
\]

For the risk, according to Markowitz, the main indicator is the variance, in case of 2 assets:

\[
\text{Var} = \text{var}(R) = \text{var}(\sum_{i=1}^{n} w_i R_i)
\]

\[
= \sum_{i=1}^{n} \sum_{j=1}^{n} \text{COV}(w_i * R_i, w_j * R_j)
\]

This leads to the equation following:

\[
\text{Var} = \text{COV} * w * w
\]

\( \text{COV} \) represents the covariance matrix of assets:

\[
\text{COV} = \begin{bmatrix}
\text{COV}(R1, R1) & \ldots & \text{COV}(R1, RN) \\
\vdots & \ddots & \vdots \\
\text{COV}(RN, R1) & \ldots & \text{COV}(RN, RN)
\end{bmatrix}
\]

Where the covariance between the return of each two assets can simply be calculated as:

\[
\text{COV}(i,j) = \sigma_{i,j} = \frac{\sum (i_j - \overline{i})(j_j - \overline{j})}{n-1}
\]

In case of 2 assets, the equation is simplified to be:
\[
\text{Var}(R) = \sum_{i=1}^{n} w_i^2 \rho_i^2 + 2 \sum_{i=1}^{n} w_i w_j \sigma_i \sigma_j \rho_{ij}
\]

To make the portfolio efficient, one needs to choose the weights such as to get the smallest variance of the rate return, given a fixed expected rate of return, or get the highest rate of return given a fixed variance.

\(\rho_{ij}\) represents the correlation between the two assets. The formula for the correlation is the following for two given assets i,j:

\[
\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}
\]

There are two extreme possible values for it:

- \(\rho_{ij} = 1\) which represents a perfect positive correlation, in which case the portfolio variance will be at its maximum since all terms are positive.
- \(\rho_{ij} = -1\) which represents a perfect negative correlation, in which case the portfolio variance will be at its minimum since all other terms remain positive.

It is then clear that the correlation has an important impact on the portfolio variance, meaning that the investor must choose assets that are more likely to be negatively correlated to minimize the risk.

### 7.2 Solution

These are constrained optimization problems and can be solved using Lagrange multipliers \(\mu_1\) and \(\mu_2\).

As explained in the previous section, the goal is to minimize the following equation:

\[
\text{Var}(R) = w_i \ast \text{COV} \ast w
\]

The constraints are:

\[
R = \sum_{i=1}^{n} w_i R_i
\]
\[ \sum_{i=1}^{n} w_i = 1 \]

\[ L(w, \mu_1, \mu_2) = w_i \cdot COV \cdot w - \mu_1 (w_i \cdot f - 1) - \mu_2 (w_i \cdot R_i - R) \]

\( f \) is the vector defined as [1,1,...,1], 1 is repeated \( n \) times which is the number of stocks considered. The two Lagrange multipliers account for the constraints.

The next step in the Lagrangian development is to find the gradients with respect to \( w_i \) for each side of the equation and solve the following system:

\[ \nabla_w L(w, \mu_1, \mu_2) = 2COV \cdot w - \mu_1 \cdot f - \mu_2 \cdot R = 0 \]

\[ \frac{\partial L(w, \mu_1, \mu_2)}{\partial \mu_1} = w_i \cdot f - 1 = 0 \]

\[ \frac{\partial L(w, \mu_1, \mu_2)}{\partial \mu_2} = w_i \cdot R_i - R = 0 \]

The solution of the system is the following:

\[ \begin{bmatrix} R_i \cdot COV^{-1} \cdot f & R_i \cdot COV^{-1} \cdot f \\ R_i \cdot COV^{-1} \cdot f & R_i \cdot COV^{-1} \cdot R \end{bmatrix} \cdot \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = 2 \cdot \begin{bmatrix} 1 \\ R_i \end{bmatrix} \]

To simplify the expression, we will create the variable X, where:

\[ X = \begin{bmatrix} R_i \cdot COV^{-1} \cdot f & R_i \cdot COV^{-1} \cdot f \\ R_i \cdot COV^{-1} \cdot f & R_i \cdot COV^{-1} \cdot R \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} \]

Giving:

\[ X \cdot \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = 2 \cdot \begin{bmatrix} 1 \\ R_i \end{bmatrix} \]

This system can only generate one solution if the determinant of \( X \) is not equal to 0.
The Lagrangian multipliers will be found using the following system:

\[
\begin{bmatrix}
\mu_1 \\
\mu_2
\end{bmatrix} = \frac{1}{\text{det}(X)} \begin{bmatrix}
R_i \ast COV^{-1} \ast R & -R_i \ast COV^{-1} \ast f \\
R_i \ast COV^{-1} \ast f & m_i \ast COV^{-1} \ast f
\end{bmatrix} * 2 * \begin{bmatrix} 1 \\
R_i
\end{bmatrix}
\]

\[
\begin{bmatrix}
\mu_1 \\
\mu_2
\end{bmatrix} = \frac{2}{\text{det}(X)} \begin{bmatrix}
R_i \ast COV^{-1} \ast R & -R_i \ast COV^{-1} \ast f \ast R_i \\
R_i \ast COV^{-1} \ast f + f_i \ast COV^{-1} \ast f \ast R_i
\end{bmatrix}
\]

At this point of development, the unknown is vector of weights, while the Lagrangian multipliers have been found. The weights are to be found through the solutions of the below equation:

\[2COV \ast w - \mu_1 \ast f - \mu_2 \ast R = 0\]

From which we get:

\[w = \frac{1}{2} \ast COV^{-1} \ast [\mu_1 \, \mu_2] \ast \begin{bmatrix} f \\
R
\end{bmatrix}\]

Finally, the weight vector is:

\[w = \frac{1}{2} COV^{-1} \ast \frac{2}{\text{det}(X)} \begin{bmatrix}
R_i \ast COV^{-1} \ast R - R_i \ast COV^{-1} \ast f \ast R_i \\
R_i \ast COV^{-1} \ast R + R_i \ast COV^{-1} \ast f \ast R_i
\end{bmatrix} \ast \begin{bmatrix} f \\
R
\end{bmatrix}\]

8. The efficient Frontier

Usually, the investor is not satisfied with one optimal portfolio combination. The efficient frontier presents a set of multiple combinations of weights with corresponding risk and return. It is the basis of the theory presented by Markowitz which is the portfolio theory. The efficient frontier is plotted graphically where the x-axis represents the risk (standard deviation) and the y-axis represents the return. It represents the set of portfolios that maximize returns for a certain risk. Returns depend on the portfolio combination, and the standard deviation is the synonym of the risk. The perfect situation for an investor is a portfolio offering high returns with a low
standard deviation. If this is successful, we can see it on the graph parallel with the efficient frontier line. The key of the optimization process if diversification that results in a curve on the graph, which means that the relationship between assets is not linear, resulting in a minimized risk.

9. Short Sell
The assumption in this model is that short selling is allowed. Short sell is a position that will allow us to have negative weights, meaning borrowing of shares. Most of the time, the lender of shares is the broker for investors. The investor usually agrees on an interest rate to be paid and invests the shares borrowed. By the end, the investor buys back the stocks from the market hoping that their price has depreciated and returns them back to the lender. The profit only exists if the stocks’ price decline, the investor will lose in case the stocks’ price rises. The main reasons behind a short sell position is speculation coming from a hidden information about the company’s future, or to minimize a risk by hedging.
10. RStudio & Functions used

The data concerning historical prices of 255 days was extracted directly from the yahoo finance website. The focus is on three main group of assets: high risk, moderate risk and low risk. Each group contains three different types of corporate stocks in diversified fields.

The code on Rstudio was implemented by respecting the steps of the mathematical development to get the final weights and the portfolio risk. The functions used are the following:

- `getSymbols()`: function used to load data, in this case used to extract data from Yahoo Finance.
- `cbind()`: combines vectors of data into one set.
- `calculate.returns()`: calculates the returns of a stock by calculating the difference between two consecutive stock prices divided by the first price.
- `na.omit()`: function used to get rid of all non available values.
- `mean()`: calculate the mean of a set of values, in this case it is used to calculate the mean of returns.
- `cov()`: calculates the covariance between data, in this case used to calculate the covariance between assets that will be put into a covariance matrix.
- `solve()`: used to solve a system of equations, in this case used to solve the covariance matrix joined with the constraints to get the weights.

First, the data for each asset was extracted, then the price chosen is the closing price which was combined for all three assets into one set. The second step is to calculate the mean of returns and create the covariance matrix that will be later on added to the constraints vector. The latter matrix will be solved by presenting a required rate of return by the investor to get the final optimal weights.
11. Model Implementation

11.1 Case 1: Low Risk Assets

The first scenario is choosing a portfolio containing three low risk assets from the Casablanca Stock Exchange website. The choice was done based on the market capitalization by taking the companies that have the highest volumes of daily trades, as well as a low volatility based on the chart generated by CSE. In this case, the companies selected are:

- Ittisalat Al-Maghrib (IAM.PA)
- Lafarge HolcimMaroc (LHN.VX)
- Attijari Wafabank (AWB)

![Volatility & Market Capitalization of IAM](image)

*Figure 3: Volatility & Market Capitalization of IAM*

From the figure above representing the volatility and market volume capitalization of Ittisalat Al-Maghrib that generally there are very small fluctuations in the daily stock market which provides the investor with stability making IAM’s stocks more desirable. Moreover, we can notice that that the market volume of daily traded stocks is high, meaning that the stocks are very liquid. These factors make IAM’s stock of low risk. [6]
The graph above represents the volatility and liquidity of Attijariwafa Bank’s stocks. We can observe that lately the stocks have stabilized and their liquidity increased sharply. This means that the stocks will be desirable by the investors in the future. This makes AWB’s stocks of low risk. [6]
For Lafarge, the main reason it is included with the low risk assets is the very high market capitalization increasing its liquidity, meaning that even if there’s a price fluctuation, stocks are traded on a daily basis which will lower the impact of the volatility. [6]

First, the data was extracted from Yahoo Finance for the three assets for the year of 2018, which generated 255 entries. Then, the return on revenue was calculated for the three assets followed by the return matrix. Using the latter, the Lagrangian matrix was generated which resulted in the weights to be allotted for each asset. The code is shown in APPENDIX A.

```r
> # the optimal weights
> w <- covInv%*(u[1]*v+u[2]*r)/2
> W

IAM.PA.Close 0.79872164
AWB.Close 0.08851145
LHN.VX.Close 0.11275590
```

Figure 6: Scenario 1 weights

As shown in the figure above, 79.87% of the capital is to be invested in Ittisalat Al-Maghrib, 8.85% is to be invested in Attijari Wafabank and 11.27% in Lafarge HolcimMaroc.

The required rate of return by the investor is 0.1%. In this case, the predicted risk of this portfolio will be around 0.8% as shown in the figure below.

```r
> # corresponding portfolio risk
> riskp<0
> riskp<-t(w)%*%cov%*%w
> sqrt(riskp)

[1,] 0.008721684
```

Figure 7: Scenario 1 risk

The efficient frontier of this portfolio was plotted to observe the patterns of the different weight combinations of the three above assets in case the investor wants to explore further portfolios. The graph represents all the optimal solutions.
11.2 Case 2: Moderate Risk Assets

For the moderate risk assets, they are characterized with a moderate volatility of prices, however they aren’t traded on a daily basis which makes their liquidity ratio lower than what an investor would prefer. In this case, the choice fell on:

- ALLIANCES (ARLP)
- AFRIQUIA GAZ (AFH)
- RISMA (AC.PA)

This was through an analysis of the following volatility charts and market capitalizations:
Afriquia Gaz’ stocks are considered to be of moderate risk. As shown in the figure above, it is characterized with a moderate market volume and moderate volatility, meaning that when investing in Atlanta’s stocks, the risk exists but is not high, but rather moderate. [6]

For Alliances, we can see in the figure shown that although they knew a set back and their prices stock decreases, the market volume remained high. This classifies Alliances within the moderate assets. [6]
It is clear that Risma has a pretty stable stock price with very small fluctuation especially lately, however their marker volume is not stable and is not very high. Due to this, Risma’s stocks are of moderate risk. [6]

The same as for the first scenario, the data extraction was done through Yahoo Finance for the 2018 year. The next step was to calculate the return on revenue and create a covariance matrix to calculate the weights. The code is shown in APPENDIX B. The following weights were the result:

```
> w<- cov(b$m%*%u[1]%*%f+u[2]%*r)/2
> w

[,1]
ARLP.Close   0.5102893
AFH.Close    -0.4451444
AC.PA.Close   0.9348551
```

*Figure 12: Scenario 2 weights*

We can notice that in this case, a negative weight was generated because short selling is allowed. The weights were distributed as follow; 51% to be allocated for ALLIANCES, a short sell of 44% for AFRIQUIA GAZ, and 93% for RISMA.

The required rate of return by the investor is around 0.1%, and the risk is 2.2% which can be considered as low as shown below:

```
> #corresponding portfolio risk
> riskp<-0
> riskp<-t(w)%*%cov%*%w
> sqrt(riskp)

[,1] 0.02249618
```

*Figure 13: Scenario 2 risk*
The following figure represents a set of optimal solutions plotted according to the expected return and risk for a combination of ALLIANCES, AFRIQUIA GAZ and RISMA.

11.3 Case 3: High Risk Assets
The high risk assets are not frequently traded and have a high volatility, which lowers their rate of return. Investors tend to stay away from these kind of assets. From the CSE website charts analysis shown below, the three high risk assets are:

- TOTAL MAROC
- ATLANTA
- M2M GROUP
In the figure above, M2M Group’s market capitalization and price volatility are graphed. It is clear that the company has some liquidity issues, however it is considered as volatile on the level of stocks’ prices. This classifies M2M in the high risk assets. [6]

In the chart above, TOTAL Maroc is characterized with a high market volume and a volatile stock price. It is classified within the high risk assets. [6]
Atlanta in the figure above shows a very small fluctuation in the stock prices and generally low volumes of shares traded. Due to these factors, Atlanta’s stocks are considered to be of high risk. [6]

As it is the case for the previous scenarios, the data was loaded from Yahoo Finance into Rstudio using the getSymbols() function. After running the code shown in APPENDIX C, the following weights were generated:

```r
> w <- cov1mv%*(u[1]+u[2])*r)/2
> w

[,1]
TOT.Close 0.53283758
M2M.F.Close 0.05660281
ATG.V.Close 0.41053960
```

The weights were distributed as follows: 53% to be allocated for Total Maroc, 5.6% for M2M GROUP, and 41% for Atlanta.

The required rate of return by the investor is around 0.1%, and risk in the case of this portfolio is around 2.5% as shown below.
For the set of optimal solutions, the efficient frontier below was plotted as a combination of different risks and returns for Atlanta, Total Maroc and M2M Group.

11.4 Case 4: Diversified Assets
The aim of this scenario is to verify whether having a well diversified group of stocks in different fields will indeed minimize the risk for a given required return. Diversification can be defined as a technique mainly employed to risk minimizing among different financial assets. Its goal is also to provide the investor with the greatest return given the same starting capital. For most financial analyst, diversification is the most important part of investment.
In this case, the idea was to choose stocks from different sectors of the Moroccan market. The companies chosen are the following:

- CAPGEMINI
- ATOS
- IAM
- AWB
- LAFARGE
- COSUMAR
- ORACLE
- ILIAD
- ACCOR
- CARREFOUR

The data was extracted from Yahoo Finance and the covariance matrix was calculated along with the constraints. The code is shown in APPENDIX D. The weights generated are the following:

```
> w<-CovInv%*%(u[1]*f+u[2]*r)/2
> w 

[,1]
IAM.PA.Close  0.37618417
AWB.Close     0.07724327
LHN.VX.Close  -0.24642601
CA.PA.Close   0.08426822
ATG.V.Close   -0.01205332
AC.PA.Close   0.22608875
TLD.PA.Close  0.21315851
ORA.PA.Close  0.06485784
ATO.PA.Close  0.17705792
CAP.PA.Close  0.03961860
```

*Figure 21: Scenario 4 weights*

We can notice from the figure above that in general the weights allocated are very small to minimize the risk and diversify the investment.

The corresponding risk for this weights combination turned out to be very small at about 0.04%. This confirms the theory that in general the more assets in different fields we have, the less risk the portfolio will generate.

```
> #corresponding portfolio risk
> riskp<-0
> riskp<-t(w)%*%cov%*%w
> sqrt(riskp)

[,1] 0.004627334
```

*Figure 22: Scenario 4 risk*
As it is the case for the other scenarios, a graph of the efficient frontier was generated to give the investor more freedom to choose the level of risk and return wanted.

![Efficient Frontier Graph](image)

*Figure 23: Scenario 4 efficient frontier*

12. **Discussion:**

From the four scenarios above, we can notice that the risk arises by going from low risk to moderate to high risk assets. The first scenario gives us a risk of 0.8%, while the second of moderate risk assets presents a risk of 22% and finally the high risk assets have a portfolio risk of 25%. This aligns with the fact that the classification of assets’ risk can be done through observing the volatility and liquidity graphs. However, the fourth scenario is the best one because the assets chosen are well diversified. Although, we didn’t take into consideration the volatility and liquidity, the simple fact that there are 10 different stocks from companies
operating in different fields presents a buffer for the risk. In this case, the risk was only 0.04% which is the lowest of all scenarios.

13. Real Life Simulation & Comparison

13.1 Les Champions de La Bourse

The CSE website offers a real life simulation game of investment portfolios. The "stock market" CdB is purely educational in nature and aims to familiarize participants of the Moroccan stock market. Each participant must manage a notional amount of MAD 1,000,000 (MAD) which must be invested in the stock market.

Each participant in the Champions de La Bourse starts the game with a fictitious initial sum of 1,000,000 dirhams (MAD) that it will be able to invest virtually on the market of the shares listed on the Stock Exchange Casablanca.

This amount is increased by a virtual amount of 300 dh per person invited by the participant. He is added to the account of the participant only after confirmation of the definitive registration of the person invited, in accordance with Article 3 of this Regulation. In addition, the total additional stake participant may not exceed 1500 DHS (The participant may invite as many moreover, only 5 guests will be taken into account). If the person invited to participate is already registered or invited by a person, the participant is not entitled to an additional bet.

The participant using his login code and his password via the Internet may achieve:
- acquisitions of the securities of his choice,
- sales of the securities held in his portfolio,
- cancellations or modification of orders,
- consultations of his portfolio.

The information on the courses, which is made available to the participants is in time real. Transactions are based on these prices to process orders.

In the event of an error on the part of the participant when entering or searching, a signal remains visible on the screen, as long as the error is not corrected by the participant.
The structure of the portfolio must respect at the end of each session the following conditions:

- **Rule 1:** 10% limit: A portfolio line must not exceed 10% of the size of the wallet.

- **Rule 2:** The number of transactions per participant is unlimited, but the maximum quantity exchanged during a day must not exceed the average daily quantity (cash set by the game organizers).

- **Rule 3:** Cash / Security Availability at Run Time: Purchase Orders will not be issued executed only up to: (cash balance of the portfolio of a participant arrested the day before increased or decreased daily trades and pending purchase orders). As for orders sales, they will only be executed in proportion to the quantity of securities available in the wallet.

- **Rule 4:** For limit orders entered, the price mentioned on the order cannot be greater than or less than 18% of the daily reference rate.

### 13.2 Low risk scenario simulation

Due to the first rule of the CdB website, only 10% of the initial capital can be invested in one set of stocks of the same company. The simulation was done as follow:

- 98 978,31 was invested in 670 stocks of Ittisalat Al Maghrib.
- 11 947,29 was invested in 8 stocks of LafargeHolcim Maroc.
- 10 209,61 was invested in 24 stocks of Attijari Wafa Bank.

The total amount invested is 119 190dhs meaning 11,94% of the initial capital. Of the amount invested, 79,9% was allocated to IAM, 8,8% to AWB and 11% to Lafarge.

This combination of investments has a required return of 0.1% and a risk of 0.8%, which is a relatively low risk.
When running the real life simulation for several times and different periods, the following results are obtained in the following way:

The website also generates graphs of the evolution of the portfolio as shown below:

The required return set by the investor is 0.1%, which is the average return on portfolios in Morocco. However, while running the simulation 5 times for different periods and the same amounts, the average return was a loss of 0.02% of the capital.
This loss shows that even though the model presented a small risk of 0.08%, there are some risks that can’t be diversified. In this case, the lowest risk asset which is IAM’s stocks turned out to provide the lowest return, even though it has a high market capitalization and low volatility.

The risk we minimized is this case is the one related to one company or one industry. However, the systematic risk related to the whole market cannot be diversified, it is referred to as “Market Risk”. It is unpredictable and cannot be fully avoided. It can be only buffered by investing in different industries, the ideal would be to invest in stocks of companies operating in different countries. And even in this case, the systematic risk is not completely eliminated. The proof is the financial crisis that became an economic crisis in 2008, and that impacted heavily all industries and most countries around the world.

12.3 Comparison

Today, the Moroccan investor makes the investment decision only based on return. To test this method, we combined the results of the simulations and ran a linear regression. It resulted in the following:

<table>
<thead>
<tr>
<th>SUMMARY OUTPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression Statistics</td>
</tr>
<tr>
<td>Multiple R</td>
</tr>
<tr>
<td>R Square</td>
</tr>
<tr>
<td>Adjusted R Square</td>
</tr>
<tr>
<td>Standard Error</td>
</tr>
<tr>
<td>Observations</td>
</tr>
</tbody>
</table>

![Figure 26: Regression Model](image)

The model used by the Moroccan investor results in an R² of 29% which is very low, meaning that the model followed of only relying on return isn’t reliable. However, the Lagrangian model
presents a minimized risk of less than 10% most of the time for low risk assets. Moreover, the model of this project gives the investor the freedom to choose the required return they expect. Finally, the model gives multiple set of portfolio combinations for the investor to choose from depending on the risk and return wanted.

### 14. STEEPLE Analysis

#### 14.1 Social Aspect
The social aspect of this project is to push investors from the Moroccan society to invest in the stock market. This model shows investors that it is easy to make decisions on which stocks to invest in, as well as how to minimize the risk. Encouraging investment means developing the economy that leads more often than not to the development of the Moroccan society, creating more job opportunities and bringing big companies to small investors.

#### 14.2 Technological Aspect
This capstone project was developed using R language to develop the Lagrange mathematical model, which means that computer science, engineering, mathematics and finance can be joined to develop predictions and make the whole investment process easier. In this case, R language wasn’t only used to upload the data directly and accurately from Yahoo Finance, but also to get the weights for each asset. Moreover, the model can be adapted for any number of assets.

#### 14.3 Environmental Aspect
The project has no direct environmental aspect. However, today, the number of environmental caring companies in Morocco is increasing sharply. By encouraging investors to buy diversified stocks, some must be of companies that have environmental impact such as companies that sell solar panels.

#### 14.4 Economic Aspect
The economic impact is the biggest in this project. Pushing Moroccans to invest is directly developing the Moroccan economy. It is helping investors take informed and wise decisions
about their investments to avoid loss and bring up the economy for even more investment, as well as increasing the Moroccan market’s liquidity.

14.5 Political & Legal Aspect
Concerning the stock market in Morocco, the rules overseeing it are made to suit Moroccan people's needs and urge them to invest to increase the liquidity. The standards are made to confine the danger of losing the capital however the downside is that one can't receive a tremendous benefit in return. With regards to insider trading, the law is as not really applied and we can see it from the variance that the market knows before openly reporting significant occasions.

14.6 Ethical Aspect
This capstone project respects all the ethical rules. The use of all sources was legal and cited. Furthermore, the data used is open from Yahoo Finance. And finally, Rstudio which is the software used to build the model is free and open source.

15. Challenges & Future Work

Investment analyst define the Moroccan market as being illiquid. For this reason, one of the biggest challenges of this project was to find matching trading days for all selected stocks of each scenario. Furthermore, before starting the work on the model, the difficulty was to understand the whereabouts of the market, it’s volatility and the models used, if there are any. However, very few scholarly articles address getting the data and eliminating the not available days was very challenging. Moreover, the extraction of data and its upload to Rstudio was a difficult task as the CSE website provides files that are not compatible with the software.

There is a lot of work that can be done in the Moroccan market, whether it is to develop the liquidity or make it more accessible to the simple investor. Today, the CSE website provides a platform for simulating portfolios that very few people know of. In the future, to make it more
accessible and easy, the CSE can develop an application for IOS and Android. Moroccans are more into technology and applications now than they have ever been. An application that simulate real life investments can really encourage people to invest more. The application can include the model developed in this project to help the user make an informed decision depending on the risk they’re willing to take and the return required.

**Conclusion**

This capstone project provides the Moroccan investor with a simple method to invest choosing their required return and risk, while keeping in mind that the most important feature is diversification. The project uses a combination of mathematics, computer science and finance. Many publicly listed Moroccan companies have been studied in order to find the best portfolios to minimize the risk, finally they were divided into three groups of risk; low, moderate and high level risk. The model uses Lagrange multipliers to solve the optimization problem. The predictions were very close to reality when comparing them with real life simulations. However, market risk should also be taken into consideration meaning that an error of absolute zero cannot be obtained. The model is explained in such a way that can allow to people with no knowledge in the field to understand and implement it. For future work, developing an application that can directly compare real life simulations with the model can bring a very important added value to the Moroccan market.
Bibliography


APPENDIX A

CODE FOR LOW RISK LEVEL ASSETS

> #Reading the data
> #Number of assets considered is N
> #Number of daily stocks considered is D
> library(quantmod)
> library(PortfolioAnalytics)
> library(PerformanceAnalytics)
> library(zoo)
> library(plotly)
> #LAM <- getSymbols(Symbol="LAM.PA", from = "2018-01-01", to = "2019-01-01", periodicity = "daily", src = "yahoo")
> #LAM2 <- LAM.PA$LAM.PA.Close
> #ATTIIAR11 <- getSymbols(Symbol="AWB", from = "2018-01-01", to = "2019-01-01", periodicity = "daily", src = "yahoo")
> #ATTIIAR12 <- AWB$AWB.Close
> #LAFARGE12 <- getSymbols(Symbol="LHNVX", from = "2018-01-01", to = "2019-01-01", periodicity = "daily", src = "yahoo")
> #LAFARGE2 <- LHNVX$LHNVX.Close
> #data3 <- chull (LAM2,ATTIIAR12,LAFARGE2)
> D=255
> N=3
> #Defining the RoR function
>
> #Creating the RoR matrix
> returns.data <- CalculateReturns(data3)
> returns.data <- na.omit(returns.data)
> #creating the expected return vector
> r<-c(rep(0,N))
> for (j in 1:N) {
>   r[j]<-mean(returns.data[,j])
> }
> #The covariance matrix
> Cov<-cov(returns.data)
>
> #Lagrangian method
> #requiredreturn is portfolio return given by the in vector
> #u is the vector representing the Lagrange multipliers
> requiredreturn=0.001
> w<-c(rep(0,N))
> CovInt<-solve(Cov)
> X=matrix(c(rep(0.4),rep(0.4)),nrow=2)
> f=c(c(rep(LN)),
> x11<-X[1,1]<-r%*%CovInt%*%f
> x12<-x21<-X[2,2]<-r%*%CovInt%*%f
> x22<-X[2,2]<-r%*%CovInt%*%f
> b=c(2,2*requiredreturn)
> u<-solve(X,b)
> #the optimal weights
> w<-CovInt%*%u[1]f+u[2]*r/2
> w
> [1]
> LAM.PA.Close 0.79873264
> AWB.Close 0.08851145
> LHNVX.Close 0.11275590
> #corresponding portfolio risk
> risk <- 0
> risk <- t(w)%*%Cov%*%t(w)
> sqrt(risk)
> [1] 0.0068721684
> #Getting the frontier
> #max is the maximum return considered
> #M is the number of portfolio in the return
>
> risk2=c(rep(0,Z))
> w2<-matrix(c(rep(0.3*Z),rep(0.7*Z)),nrow=3)
> u2<-matrix(c(rep(0.2*Z)),nrow=2)
> b2<-matrix(c(rep(0.2*Z)),nrow=2)
> for (z in 1,Z) {
>   expectedreturn[z]<-z%*%max.Z
>   bfr[z]<-c(2,2*expectedreturn[z])
>   sol[z]<solve(X,bfr[z])
>   wfr[z]<CovInt%*%u[1,z]*bfr[z]*r/2
>   risk[z]<-sqrt(t(wfr[z])*%*%Cov%*%wfr[z])
>   }
> plot(risk2,expectedreturn)
APPENDIX B

CODE FOR LOW RISK LEVEL ASSETS

```r
> # Reading the data
> # Number of assets considered is N
> # Number of daily stocks considered
> library(quantmod)
> library(PortfolioAnalytics)
> library(PerformanceAnalytics)
> library(zoo)
> library(quantmod)
> # ALLACIES <- getSymbols(Symbol = "ARLP", from = "2018-01-01", to = "2019-01-01", periodicity = "daily", src = "yahoo")
> # ALLACIES2 <- ARLPHOLP.Close
> # AFRICAN:< getSymbols(Symbol = "AFH", from = "2018-01-01", to = "2019-01-01", periodicity = "daily", src = "yahoo")
> # RISMA <- getSymbols(Symbol = "AC,PA", from = "2018-01-01", to = "2019-01-01", periodicity = "daily", src = "yahoo")
> # RISMA2 <- AC,PA,AC,PA.Close
> # Combining the data
> data3 <- cbind(ALLACIES2, AFRICAN, RISMA2)
> N <- 255
> N = 3
> # Defining first the ROR function
> # Giving prices’ vector, the function will return the ROR
> # Creating the ROR matrix
> returns.data <- CalculateReturns(data3)
> returns.data <- na.omit(returns.data)
> # Creating the expected return vector
# r <- c(rep(QSON),)
> for (i in 1:N) {
>   r[i] <- mean(returns.data[i,])
> }
> # Computing the covariance matrix
> cov <- cov(returns.data, returns.data)
> # Lagrangean method
> # Required return is portfolio return given by the investor
> # u is the vector representing the Lagrange multipliers
> # requiredreturn = 0.001
> w <- c(rep(0.1, N))
> CovInv <- solve(cov)
> X = matrix(c(rep(0, 1)), nrow = 2)
> f = c(rep(1, N))
> x1 <- X[1, 1] <- r% CovInv% x
> x2 <- x1 <- X[2, 1] <- r% CovInv% x
> w2 <- x2 <- X[2, 2] <- r% CovInv % x
> b = (c(2, requiredreturn))
> u <- solve(X, b)
> # The optimal weights
> w = CovInv % (u[1] * f ++ u[2] * r) / 2
> w
> [1,]
> ARLPHOLP.Close 0.5102893
> AFRICAN.Close 0.4451444
> AC,PA.Close 0.0458551
> # Corresponding portfolio risk
> risk2 <- 0
> risk2 <- t(w)% CovInv% w
> sort(risk2)
> [1,] 0.0229616
> # Getting the frontier
> # Amax is the maximum return considered
> # N is the number of portfolio in the return
> Amax = 0.01
> Z = 100
> expectedreturn = c(rep(QZ, Z))
> risk2 = c(rep(QZ, Z))
> wfr = matrix(c(rep(0, 3 * Z), nrow = 3)
> wfr = matrix(c(rep(0, Z), nrow = 2)
> for (z in 1:Z) {
>   bfr[z] <- c(2, 2 * expectedreturn[2])
>   ufr[z] <- solve(X, bfr[z])
>   wfr[z] <- CovInv % (ufr[z] * ffr[z] + r)(z) / 2
>   risk2[z] <- sort(t(wfr[z]) % CovInv% wfr[z])
>   plot(risk2, expectedreturn)
```

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APPENDIX C

CODE FOR HIGH RISK LEVEL ASSETS

```r
> # Reading the data
> # Number of assets considered is N
> # Number of daily stocks considered
> library(quantmod)
> library(PortfolioAnalytics)
> library(PerformanceAnalytics)
> library(zoo)
> library(gplots)
> TOTAL <- getSymbols(Symbol="jnj", from = "2018-01-01", to = "2019-01-01", periodicity = "daily", src = "yahoo")
> TOTAL2 <- TOTAL[,3]
> M2M <- getSymbols(Symbol="M2M1", from = "2018-01-01", to = "2019-01-01", periodicity = "daily", src = "yahoo")
> M2M2 <- M2M$M2M1.Close
> ATLANTA <- getSymbols(Symbol="AG", from = "2018-01-01", to = "2019-01-01", periodicity = "daily", src = "yahoo")
> ATLANTA2 <- ATLANTA[,3]
> # Combining the data
> data3 <- cbind(TOTAL2,M2M2,ATLANTA2)
> D=255
> N=3
> # Defining first the R0 function
>
> # Giving prices' vector, the function will return the ROR
> # Creating the ROR matrix
> returns.data <- calculateReturns(data3)
> returns.data <- na.omit(returns.data)
> # Creating the expected return vector
> r<-c(rep(0,D))
> for (i in 1:N) {
+ r[i]<-mean(returns.data[,i])
+ }
> # Computing the covariance matrix
> Cov=cov(returns.data,returns.data)
> # Lagrangian method
> # requiredReturn is portfolio return given by the investor
> # w is the vector representing the lagrange multipliers
> requiredReturn=0.001
> w<-c(rep(0,N))
> CovInv<-solve(Cov)
> X=matrix(c(rep(0,4)),ncol=2)
> f<-c(rep(1,N))
> x1<-x[1,1]<-f%*%CovInv%*%X
> x2<-x[2,1]<-f%*%CovInv%*%X
> b<-c(2,2*requiredReturn)
> u<-solve(X,b)
> # The optimal weights
> w<-cov.eig%*%(u[1]%*%u[2])^1/2
> w
>
> [1] 0.53285758
> M2M1.Close 0.05660281
> AG.Close 0.4105960
> # Corresponding portfolio risk
> riskp<-0
> riskp<-t(w)%*%Cov%*%w
> sqrt(c(riskp)
> [1,] 0.2576026
> # Getting the frontier
> # rmax is the maximum return considered
> # m is the number of portfolio in the return
> rmax=0.01
> expectedReturn=c(rep(0,N))
>
> # Risk2=c(rep(0,N))
> w2<-matrix(c(rep(0,3*2)),ncol=3)
> # Risk2=c(rep(0,2*2))
> for (z in 1:D) {
+ expectedReturn[z]<-z*rmax/Z
+ w[c(z,2*expectedReturn[z])]<-c(2,2*expectedReturn[z])
+ w[c(z,2)]<-solve(2,b[c(z,2)])
+ w[c(z,1)]<-cov.eig%*%(u[1]%*%w[c(z,2)]%*%u[1])
+ }
> plot(risk2,expectedReturn)
```

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APPENDIX D

CODE FOR DIVERSIFIED ASSETS

# Reading the data
# Number of assets considered is N
# Number of daily stocks considered
library(quanteda)
library(PortfolioAnalytics)
library(PerformanceAnalytics)
library(zoo)
library(plots)

# Define symbols
IAW <- getSymbols(Symbol = "IAW.PA", from = "2018-01-01")
IAM2 <- IAW[, "Close"
ATTIJARI <- getSymbols(Symbol = "ATTIJARI", from = "2018-01-01")
ATAV <- ATTIJARI[, "Close"
LAFARGE <- getSymbols(Symbol = "LAFARGE", from = "2018-01-01")
LAFARGE2 <- LAFARGE[, "Close"
CARDEFOUR <- getSymbols(Symbol = "CARDEFOUR", from = "2018-01-01")
CARDEFOUR2 <- CARDEFOUR[, "Close"

# Define symbols
ATLANTA <- getSymbols(Symbol = "ATLANTA", from = "2018-01-01")
ATLANTA2 <- ATLANTA[, "Close"

# Define symbols
RISMA <- getSymbols(Symbol = "RISMA", from = "2018-01-01")
RISMA2 <- RISMA[, "Close"

# Define symbols
ILDA <- getSymbols(Symbol = "ILDA", from = "2018-01-01")
ILDA2 <- ILDA[, "Close"

# Define symbols
ORANGE <- getSymbols(Symbol = "ORANGE", from = "2018-01-01")
ORANGE2 <- ORANGE[, "Close"

# Reading the data
# Number of assets considered is N
# Number of daily stocks considered

# Library functions
library(quanteda)
library(PortfolioAnalytics)
library(PerformanceAnalytics)
library(zoo)
library(plots)

# Define symbols
IAW <- getSymbols(Symbol = "IAW.PA", from = "2018-01-01")
IAM2 <- IAW[, "Close"
ATTIJARI <- getSymbols(Symbol = "ATTIJARI", from = "2018-01-01")
ATAV <- ATTIJARI[, "Close"
LAFARGE <- getSymbols(Symbol = "LAFARGE", from = "2018-01-01")
LAFARGE2 <- LAFARGE[, "Close"
CARDEFOUR <- getSymbols(Symbol = "CARDEFOUR", from = "2018-01-01")
CARDEFOUR2 <- CARDEFOUR[, "Close"

# Define symbols
ATLANTA <- getSymbols(Symbol = "ATLANTA", from = "2018-01-01")
ATLANTA2 <- ATLANTA[, "Close"

# Define symbols
RISMA <- getSymbols(Symbol = "RISMA", from = "2018-01-01")
RISMA2 <- RISMA[, "Close"

# Define symbols
ILDA <- getSymbols(Symbol = "ILDA", from = "2018-01-01")
ILDA2 <- ILDA[, "Close"

# Define symbols
ORANGE <- getSymbols(Symbol = "ORANGE", from = "2018-01-01")
ORANGE2 <- ORANGE[, "Close"

# Calculating the covariance matrix
returns.data <- cbind(IAW, IAM2, ATTIJARI, LAFARGE, CARDEFOUR, ATLANTA, RISMA, ILDA, ORANGE)
returns.data <- na.omit(returns.data)
covmatrix <- cov(returns.data)
covmatrix

# Using the covariance matrix
returns.data <- cbind(IAW, IAM2, ATTIJARI, LAFARGE, CARDEFOUR, ATLANTA, RISMA, ILDA, ORANGE)
returns.data <- na.omit(returns.data)
covmatrix <- cov(returns.data)
covmatrix
APPENDIX E

REAL LIFE SIMULATIONS
## APPENDIX F

### SAMPLE DATA

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