INVESTMENT STRATEGIES: PORTFOLIO OPTIMIZATION & RISK MANAGEMENT WITH R LANGUAGE

FINAL REPORT

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INVESTMENTS: PORTFOLIO OPTIMIZATION & RISK MGT W/ R LANGUAGE.

Capstone Report

Student Statement:
This project intends to exhibit my own research and work under the guidance and rightful supervision of Dr. Laayouni Lahcen. Ethics and public safety have been taken into consideration and any information used has been attributed to its rightful author. The method and design used states the different assumptions and riskiness one should take into consideration beforehand.

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Approved by the Supervisor(s)

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Dr. L. Laayouni
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ABSTRACT

Many investors, also known to companies as shareholders, seek to maximize their wealth by taking the minimum risk. For this reason, this capstone project aims to do exactly that by encouraging shareholders making an investment decision through a selection of portfolios with different risk tolerance (low, moderate, and high). This project intends to help investors in the American Stock Market (NASDAQ, NYSE…etc.) invest and get the return desired for their tolerance of risk. It implements the Mathematical Model of Lagrange Multipliers and relies on Markowitz’s portfolio theory. Besides the implemented coding that facilitates the whole procedure of optimization, this project provides a real-life simulation. In addition to that, in order to evaluate the added value of the project, a STEEPLE analysis has been done. Future work is also discussed in order to help the reader go beyond what has been done and have an idea about additional work or options that he can opt for.

Keywords: STEEPLE analysis, Quantitative Analysis, Portfolio optimization, NYSE, NASDAQ, Markowitz, Lagrange Multiplier, Modern Portfolio Theory
INTRODUCTION

Context:
Investments constitute a substantial part of our life. On a macro scale, they help us achieve *economic growth* for the nation we invest in. On a micro scale, they help *increase our wealth* and live more comfortably. Many shareholders view investments as an opportunity of growing wealth at a fast pace. If done correctly, the investor may earn a fundamental amount of money thanks to their return on investment. Today, instead of investing in one financial asset, people prefer to devote their money to what we call *portfolios*; a set of financial assets (Stocks, bonds, options…etc.). Rational investors make sure their portfolios are *diversified* in order to minimize the *unsystematic risk*. The big question remains in the *assets’ allocation* which is defined as “An investment strategy that aims to balance risk and reward by apportioning a portfolio’s assets according to an individual’s goals, risk tolerance and investment horizon” [1]. Investors long for the best *combinations* possible of investments, which go hand in hand with their preferences. In other words, they are always in the quest of what could be the best outcome.

In a world like ours, it is almost impossible to count for all the different parameters that could affect the market. Markets all around the globe are in a *constant change* as they adapt to all kinds of *events* that are taking place every second. Taking as *an example* the famous interview of Elon Musk with the New York Times on Friday in which he stated being overworked [2], following that specific event, the Tesla Stock went down as of *August 17*, 2018 (see figure below).

![Tesla’s Share price as of August 2018](image)

Figure 1. Tesla’s Share price as of August 2018 [2]
Because of a simple interview, the stock price went down. A potential explanation could be that investors feared the Tesla Company would not be working to its fully potential, or that the quality of its product would decline because of at that time, the company’s CEO felt unfit and overworked.

In order to overcome these kinds of situations, and take as many parameters as possible into account, mathematicians and statisticians have developed many models that could be best solve the problem of assets’ allocations. It is important to keep in mind that each model or theory presented has its own assumptions. For this reason, every single assumption should be taken into consideration before proceeding into the usage of the model proposed. Also, the investor should be able to evaluate his own risk aversion. Risk management should not be considered as a trivial matter as it is an important criterion for a proper analysis of assets’ allocation. Investing in very volatile stocks could be bad for a high-risk averse potential stockholder. Finally, usually the variables used are estimated, meaning that the models suggested by mathematicians or analysts are not absolutely true or guarantee the outcome desired. Accounting for errors is primordial and should not be neglected.
CHAPTER I

1.1 HISTORY & FUNDAMENTALS ON PORTFOLIO THEORY

Before we proceed to the explanation and the implementation of the model chosen, the understanding of the roots of portfolio theory is significant. Note; the aim behind this project is to come up with a set of financial assets that goes along with a potential’s investor preferences and risk tolerance. Preferences usually are considered as constraints, as they restrict some aspects of the model. An example of a constraint would be the openness of the investor to short selling, which later will be explained along the report. Thanks to modern portfolio theory first introduced by Harry Markowitz, today, many mathematicians and analysts were able to develop different models based on it to help optimize portfolios. In our case, we will be optimizing the portfolio by minimizing the risk for a given required rate of return from a possible investor.

A groundbreaking concept in the field of optimization of portfolios has seen the day with the Nobel Prize winner Harry Markowitz, in our days considered as the father of modern portfolio theory [3, Page 1]. The economist introduced his concept in his essay “Portfolio Selection” in 1952 and it was published in the “The Journal of Finance” [3, Page 1].

In his essay, Harry Markowitz introduced a different way of optimizing portfolios. In fact, at the time, most investors were basing their asset allocation on experience and observation. In his “Portfolio Selection” paper, the economist suggests a new way of looking at things. He claims that his theory, despite the number of assumptions, could be a potential answer to investor’s problems as it takes into account their preferences. [4, Page 77]. In addition to what was mentioned before, Markowitz does not fail to clearly introduce the concept of taking the overall risk-reward criterion of the entire portfolio instead of combining individual financial assets with appealing returns [5, Page 61]. Needless to add the fact that Markowitz theory relies heavily on a number of assumptions, which, theoretically are correct, but cannot be applied on a realistic scale due to the different uncontrollable factors. Myles E. Mangram, from the SMC University in Switzerland summarizes the different assumptions of the economist as follows:

a. “Investors are rational.” [5, Page 61].

b. “Investors are only willing to accept higher amounts of risk if they are compensated by higher expected returns.” [5, Page 61].
c. “Investors timely receive all pertinent information related to their investment decision” [5, Page 61].

d. “Investors can borrow or lend an unlimited amount of capital at a risk-free rate of interest” [5, Page 61].

e. “Markets are perfectly efficient” [5, Page 61].

f. “Markets do not include transaction costs or taxes” [5, Page 61].

g. “It is possible to select securities whose individual performance is independent of other portfolio investments” [5, Page 61].

Many of the assumptions stated above are unrealistic and inapplicable in real-life market. It is easy to refute the fact that markets are efficient, since most of the cases, they are not, and costs of transactions and taxes are always included in the fees even online. Also, some financial assets have correlations which makes them dependent on each other. In an era like ours, and with the globalization introduced, having almost no dependency between industries or securities in general is almost impossible.

The whole concept introduced by the economist was solely based on the variances and the returns of the assets/portfolio. Having the right combinations of both, could make us obtain the optimal portfolio [6].

1.2 THE AMERICAN STOCK MARKET

When talking about the American Stock Market, most people would think of the New York Stock Exchange. It is important to make the different between the American Stock Market and the different stock exchange that could exist in the same country.

Let us first define what a stock market is. According to Investopedia, “The stock market refers to the collection of markets and exchanges where regular activities of buying, selling and issuance of shares of publicly-held companies take place” [7]. On the other hand, when talking about stock exchange, it refers to a platform where people can trade financial securities like options, bonds…etc. There is a number of Stock Exchange facilities in the United States of America. Nonetheless, the most popular ones remain the NYSE (The New York Stock Exchange) and the NASDAQ (National Association of Securities Dealers Automated
Quotations) and our portfolio selection for this project will come from companies listed on those two trading facilities.

It is also important to add that the stock market is has many regulators such as the SEC (Securities and Exchange Commission) [8]. Their mission could be summarized in the three following statements: They make sure the markets are working at their full capacities; efficient. They regulate and track down fraudulent behavior such as insider trading. They make sure the investor are protected, and they ease the net capital accumulation [9].

For now, in the American Stock Exchange, the main financial securities traded remain very standard as they are mainly: Stocks, Bonds, Options, Commodities…etc. In our project, we are only going to work with stocks since it facilitates the understanding of portfolio optimization and are not as complex as other securities. However, today we hear a lot about what we call cryptocurrency trading, which have not yet been introduced in those Stock Exchanges even though they can be traded on virtual platforms such as Binance.com or Coinbase.com. In the last quarter of 2018, the CEO of NYSE has stated that despite the number of “irregularities” and freedom the world is seeing in cryptocurrency trading, it could indeed have a place in the regulated markets [10]. “Sprecher’s Intercontinental Exchange, along with Starbucks, Microsoft and BCG, is backing a new company called Bakkt that will facilitate bitcoin futures trading by the first quarter of next year.” [10]. The latter all points toward a new era of trading which includes digital currencies, and that people would undoubtedly include in their respective portfolios if they happen to be low risk averse.
CHAPTER II
STEEPLE ANALYSIS

2.1 SOCIETAL
This project constructively helps the development of the society in the sense that it attracts investors to explore potential markets, and thus encourages new start-up companies to produce more, and consequently, by increasing the amount of production, employment increases, research studies in the quest of developing new products increases and benefits the society.

2.2 TECHNICAL
As required by the instruction within the syllabus, this project use many technical tools and software such as Rstudio and VScode (Visual Studio Code) since an R implementation is done. Using a programming language facilitate the analysis of data and the model’s implementation. However, the reader or the user must be able to use it and read it, meaning that it requires some prior knowledge. That is why; a technical part dedicated solely for the explanation of the implementation of the mathematical model used is added.

2.3 ECONOMIC
Thanks to portfolio theory used in this project, many investors are now always longing for the maximization of their wealth. Because the optimization of portfolios help investors reach their goals, it encourages them to invest more in new and existing markets. Doing so in emerging countries, like Morocco or Brazil, would most likely help them advance the economy and encourage new companies to materialize. Along with the benefits that come along with the increasing wealth of the shareholders and the company, CEOs would want to eradicate the agency problem; managers would become motivated and increase their productivity (Improvement of skills and qualification, development of research and technology, use of new management techniques…etc.). All the latter would encourage the improvement and the boost of those countries economic growth.

2.4 ETHICAL
The mathematical model used in this portfolio optimization abides by the law, and does not encourage unethical behavior. For instance, some unethical managers or brokers may facilitate margin abuses such as the evasion of margin requirements imposed by the Federal Reserve Board, the SEC or the SROs, which is something that is not promoted in this implementation. In addition to that, all the information provided in this report and all the
concepts used are attributed to their respective owners. Besides, as clearly stated in the introduction, the model used in this project is reliable but may lead to incorrect or undesirable outputs as all the information or inputs used are estimated and not absolute.

2.5 POLITICAL
Portfolio optimization projects generally do not affect the country’s political ideologies in any way. However, many fiscal regulations and policies may in fact affect the decision of the investor, and such parameters must be taken into consideration by the model. In addition to that, it is important to shake the fact that political events may have major effects on the stock prices and thus shake the market. That is why, the investor is required to be aware and informed of the events because those can adjust risks and alter the results.

2.6 LEGAL
Every method used in this project is lawful and does not promote or encourage any fraudulent behavior. Front running and insider trading are not taken into consideration. Every single concept introduced is by the book.

2.7 ENVIRONMENT
The project’s model does not have any effect whatsoever on the environment per se. Since the simulations and the trading is all done virtually, nothing can affect the environment neither in a good nor in a bad way.
CHAPTER III
METODOLOGY

3.1 – IMPORTANT DEFINITIONS

3.1.1 Diversification
While investing in the American Stock Market, investors face two different kinds of risks: The systematic risk and unsystematic risk (explained later in the report). As a risk management strategy, it is possible to lower one of the two, which is the unsystematic risk. In order to lower it, investors tend to budget their money in portfolios with diversified assets. Choosing assets from different industries lowers the risk of losing money.

The rational reasoning behind that comes from the fact that if an industry is not doing well and the company, which is working within that industry, is eventually not showing a great performance, its required rate of return is not going to meet the expectations, and that could be offset by investing in companies working in other industries.

3.1.2 Risk
The definition of the risk differs from one analyst to another. Every book and every report would hold a different one. Yet, the unit of measurement is usually agreed-upon every time. Risk could be defined as the percentage amount by how much the actual return deviates from the mean return. If the dispersion is very spread out, we have high risk, thus high uncertainty, and vice versa. [10]

Risk is usually measured by calculating the variance or the standard deviation.

\[ \text{Standard Deviation} = \sqrt{\text{Variance}} = \sqrt{\sigma^2} \]

There are two types of risks: Systematic and Unsystematic Risk.

3.1.2.1 Systematic Risk
Systematic risk, also known as market risk is an undiversifiable risk. The reason behind that is due to the fact that it exhibits the volatility and fluctuations that happen in the market due to large-scale factors that are out of control such as inflation.
3.1.2.2 Unsystematic Risk

Unlike systematic risk, the unsystematic one can be diversified. The reason behind that is that this unsystematic risk happens due to microeconomic factors that can be in our control. It is a risk proper to the company you invest in. Therefore, it makes sense to think that the risk that we are talking about when investing in a portfolio is the unsystematic risk since it is the one that we are trying to minimize through diversification.

To calculate the risk (standard deviation) of the portfolio as a way of quantifying the divergence from the expected mean return, it is crucial to find the covariance between the different assets chosen in the portfolio.

**For individual Securities [i]:**

\[
\text{Variance} = \text{Variance}[\text{Return}_i] = \text{Expected}_\text{Return}[\text{Return}_i - \text{Mean}_i] = \sigma_i^2
\]

\[
\text{Standard Deviation} = \sqrt{\text{Variance}[\text{Return}_i]} = \sigma_i
\]

**For the portfolio:**

\[
\text{var}_\text{portfolio} = E[(R - E(R))^2] = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i * w_j * s_{ij} = \text{vec}_\text{weights}^T \text{Cov}\text{Matrix}w
\]

⇒ The variance of the portfolio is simply the weighted average of the variance of the individual securities.

\[
\text{Cov}\text{Matrix} =
\begin{bmatrix}
\text{cov}_{11} & \text{cov}_{12} & \ldots & \text{cov}_{1N} \\
\text{cov}_{12} & \text{cov}_{22} & \ldots & \text{cov}_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
\text{cov}_{N1} & \text{cov}_{N2} & \ldots & \text{cov}_{NN}
\end{bmatrix}
\]

where:

\[
\text{cov}_{ij} = \text{cov}_{ji} = E[(R_i - r_i) * (R_j - r_j)]
\]

\[
w = [w_1, w_2, \ldots, w_N]^T
\]


For a better understanding of the reasoning behind why the covariance matrix is of such a great importance, let us take an example of a portfolio of two assets:
Its respected variance should be calculated as follows:

$$\text{var}_p = W_a \sigma_a^2 + 2 * W_a W_b \sigma_{a,b} + W_b \sigma_b^2$$

**Remember:** \( \sigma_{a,b} = \text{correlation}_{a,b} * \sigma_a * \sigma_b \)

- \( \text{correlation}_{a,b} \neq 0 \), meaning the two financial securities behave in the same directions, and that there is a relationship between them.
- \( \text{correlation}_{a,b} = 0 \), no relationship between the two assets chosen.
- \( \text{correlation}_{a,b} \neq 0 \), meaning the two financial securities behave in the opposite directions, and that there is a relationship between them.

**Note:**
In our model, we assume that the covariance is constant over time between the different financial securities chosen. This assumption does not apply in real-life, but further work in order to overcome this restriction is discussed later on.

### 3.1.3 Return

#### 3.1.3.1 – Expected Return

When investing in a certain stock, being a shareholder makes you part of one of the owners of the company. In the case of a booming period for the company, in other words, if the company is making a lot of profit, the company could opt for the option of distributing some of it to the respective stockholder [10]. The profit the stockholder gets is often referred to as the return (Dividend yield + Capital gain/loss).

The return is an important component of our model. Predictions of the return are always uncertain. In order to compute it, we decide to rely on the following equation:

$$\text{Closing Price}_t - \text{Closing Price}_{t+1}$$

$$\text{Closing Price}_t$$

There are different ways to estimate the expected return, but the one chosen and that is implemented in this model is the one relying on the calculation of the arithmetic mean of historical returns of roughly 250 days.

$$R_{\text{periods}} = \frac{1}{n} \sum_{i=1}^{n} R_i = \frac{R_1 + R_2 + \cdots + R_n}{n}$$
For individual returns in a portfolio, the mean is: $E[R_i] = \mu_i$, with $i = 1,2,3,...,n$, with $n \in \mathbb{N}$. Meaning that $R_i$ is a representation of the expected return on each stock.

Mathematically, we consider $R_i$ as a vector.

The portfolio’s overall expected return is the following:

$$R_{[0,T]} = \sum_{i=1}^{n} \omega_i R_i,$$

with $\omega_i = \text{Weight on security } i$

or $R_p = \omega_1 R_1 + \omega_2 R_2 + \cdots + \omega_n R_n$, source “Portfolio Optimization in R” [11]

### 3.1.3.2 – Required Rate of Return

It is important to make the difference between the Expected Return and the Required Rate of Return. As demonstrated before, the Expected return is what is expected to be collected from an investment. Whereas, the required rate of return is what the investor REQUIRES before investment in a financial asset. It is the minimum value of return the investor requires so that he could consider making an investment. For instance, suppose that a person wants to invest in security A and that his required rate of return is 5%. If the expected return is 3%, the investor would definitely not invest in the latter.

### 3.1.4 Short Selling

In the American market, the investor has the advantage of doing what we call “short selling”. As defined by the CFA institute, short selling is “the practice whereby (equity and non-equity) market participants sell securities they do not own with a view of buying them back at a lower price.” [12]. When calculating the weights that the investor is advised to budget for each asset of the portfolios chosen, some weights may have negative values. Those negative values understate the fact that the investor should short-sell.

To go in more in-depth of what short-selling/shorting a stock consist in, it is a practice that requires borrowing shares from another owner and selling them with the hope that the price will go down. After doing so, the investor buys them again (at a very low price), gives them back to the respective owner, and earns the difference [13].

### 3.1.5 Long Buying (Long-Position)

Unlike the short-selling position, long buying is the activity of buying a stock with no intention whatsoever of selling it in the near future. The investor hopes that the price will go
up with time and that he/she would make a profit by doing so. In our project, the positive weights represent stocks that our model suggest the investor to long-buy instead of short-sell.

3.2 – MEAN-VARIANCE ANALYSIS

Model Used
As explained before, the method used in this situation is the mean-variance analysis, which relies on Markowitz portfolio theory. It is a method mainly used for the optimization of portfolios in the stock market. In our situation, our objective is to minimize the risk with a given required rate of return from the investor.

3.2.1 Optimization
In order to implement this model, which resides in solving a quadratic optimization problem, we must first set the objective as well as the different constraints we are taking into considerations. Mathematical equations to express our objective and the different constraints considered are expressed in our case as follows:

- **Objective:**

  \[ \text{weights}_{\text{portfolio}} = \arg \min \{ \text{var}_{\text{portfolio}} \} \]
  
  *Could also be written as:*
  \[ w = \arg \min \{ s^2 \} \]

  \[ \text{argmin} \Leftrightarrow \text{"gives a position } x_{\text{min}} \text{ at which } f \text{ is minimized"} \] [14].

- **Constraints:**

  We consider two main constraints in our model:

  - Here, short selling is allowed so we will not be concerned about the signs of the weights. However, the sum of the weights must equal one. Meaning that the **wealth is constant.**
    
    Meaning that: \( \sum_{n=1}^{N \text{(Number of Assets in portfolio)}} w_i = 1 \)

  - The expected return in our case must equal the required expected return on the portfolio by the investor.
    
    Meaning that: \( w^T \cdot r = \sum_{n=1}^{N \text{(Number of Assets in portfolio)}} W_n r_n = r_p \)
    
    With \( r = [r_1, r_2, \ldots, r_N]^T \)

Here, we are taking only two constraints to facilitate the understanding of the logic behind to the reader. Obviously, the program implemented is written in a way that it can be modified and edited according to the preference of the investor. The project does not try to orient the thinking
toward a “specific” type of preferences that is why; it has only taken into consideration the most substantial ones.

Now that the objective and the constraints has been set, we can proceed towards the finding of the solution of the system using the **Lagrangian Method of Multipliers**.

### 3.2.2 Lagrange Multiplier Method

First, it is important to define the Lagrangian:

\[
L(w, \delta_1, \delta_2) = w^T \text{cov} w - \delta_1 v - \delta_2 (w^T r - r_p)
\]

First constraint

Second constraint

\[ v = [1,1 \ldots ,1], \text{we have as many } 1 \text{'s as we have the number the number of assets.} \]

The Lagrange multipliers are:

\[
\delta = \begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix}
\]

Because in this case we only have two constraints, we have only two Lagrange Multipliers. Usually we have as many Lagrange Multipliers as we have constraints.

As a solution, we do the following:

\[ v_w L(w, \delta_1, \delta_2) = 2 * \text{covariance} * w - \delta_1 v - \delta_2 r \quad (1) \]

\[ \Rightarrow \text{Here, we perform the partial derivation with respect to the individual weights.} \]

After that, we calculate the partial derivatives with respect to the Lagrange Multipliers:

\[
\frac{\partial L(w, \delta_1, \delta_2)}{\partial \delta_1} = w^T v - 1 = 0 \quad (2)
\]

\[
\frac{\partial L(w, \delta_1, \delta_2)}{\partial \delta_2} = w^T r - \rho = 0 \quad (3)
\]

**From equation (1), we get:**

\[ w = \frac{1}{2} \text{cov}^{-1}(\delta_1 v + \delta_2 * r) \]

**From equation (2), we get:**

\[ v^T \text{cov}^{-1} v \delta_1 + r^T \text{cov}^{-1} v \delta_2 = 2 \]

**From equation (3), we get:**

\[ v^T \text{cov}^{-1} r \delta_1 + r^T \text{cov}^{-1} r \delta_2 = 2 \rho \]

Notice: \[ v^T \text{cov}^{-1} r = r^T \text{cov}^{-1} v \]
Another way to write the equations above could be the following:

\[
\begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{bmatrix}
\begin{bmatrix}
\delta_1 \\
\delta_2
\end{bmatrix}
= 2
\begin{bmatrix}
1 \\
\rho
\end{bmatrix}
\]

With \( Z = \begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{bmatrix} = \begin{bmatrix}
u^T\text{cov}^{-1}u & r^T\text{cov}^{-1}r \\
v^T\text{cov}^{-1}r & r^T\text{cov}^{-1}r
\end{bmatrix}\)

The matrix above is invertible and has a unique solution. Consequently, \( Z \)'s determinant is different from zero.

The solution can be expressed as:

\[
\begin{bmatrix}
\delta_1 \\
\delta_2
\end{bmatrix}
= \frac{1}{\det(Z)} \times \begin{bmatrix}
u^T\text{cov}^{-1}u & r^T\text{cov}^{-1}r \\
v^T\text{cov}^{-1}r & r^T\text{cov}^{-1}r
\end{bmatrix} \times 2 \times \begin{bmatrix}
1 \\
r_p
\end{bmatrix}
\]

\[
\Rightarrow \begin{bmatrix}
\delta_1 \\
\delta_2
\end{bmatrix}
= \frac{2}{\det(Z)} \times \begin{bmatrix}
u^T\text{cov}^{-1}u & r^T\text{cov}^{-1}r \\
v^T\text{cov}^{-1}r & r^T\text{cov}^{-1}r
\end{bmatrix}
\]

At this point, we have already determined the \textit{Lagrange Multipliers} for the two constraints.

It is easier to determine the weights since they are the only unknown left.

\[
2 \times \text{cov} \times w - \delta_1 \times u - \delta_2 \times r = 0
\]

\[
\Rightarrow 2 \times \text{cov} \times w - [\delta_1 \quad \delta_2] [u] = 0
\]

\[
\Rightarrow w = \frac{1}{2} \text{cov}^{-1} [\delta_1 \quad \delta_2] [u]
\]

As a result, the optimal weights' could be expressed as:

\[
w = \frac{1}{2} \frac{\text{cov}^{-1}}{\text{det}(Z)} \times \begin{bmatrix}
r^T\text{cov}^{-1}r - r^T\text{cov}^{-1}u \times r_p \\
r^T\text{cov}^{-1}u + r^T\text{cov}^{-1}u \times r_p
\end{bmatrix} [u]
\]

\[
\begin{array}{c}
2.3.3 \text{ Efficient Frontier}
\end{array}
\]

The efficient frontier is a graph that plots the different combinations of optimized portfolios for different expected portfolio returns and the standard deviation. In other words, it shows a contrast or a comparison between the different risks and the expected returns. Every single combination on the curve obtained represents actually the highest expected return possible for a known amount of standard deviation (risk of portfolio). Originally, it was referred to as “Markowitz Efficient Frontier” [5, Page 66].

The efficient frontier could help investors get an idea about the different combinations possible. The reason why this curve is so handy is because every single investor has a different amount of risk aversion (preferences) for given expected returns, and thus, they can choose the combinations they prefer following their own investment strategy.
It is important to notice that the efficient frontier highlights greatly the benefits of diversification. In fact, if a number of securities have a positive correlation (moving in the same direction) the risk is greater. The opposite is also true.

For instance, if security A is a stock in a company working in the automotive industry, and if security B is a stock in a company working in the Materials industry (Steel). It is clear that their correlation would be positive and they would both move in the same direction since steel is one of the raw materials used in the creation of cars.

3.3 ETHICS, FRAUDULENT BEHAVIOR TO AVOID ON THE TRADING FLOOR

3.3.1 Insider Trading
As discussed in the STEEPLE analysis stated above, this project does not promote any unethical/illegal behavior. In order to take advantage of the stock market, some investors tend to do insider trading. It could be defined as a practice that consists in getting confidential information on a security. Usually, according to the SEC, that material is obtained through fiduciary duty [15].

The reason why the latter is an illegal practice is due to the fact that each and every investor is supposed to have the same and equal opportunity on a given stock market. Having access to non-public information does not provide exactly that. Again, there are many nuances on how to define insider trading since some part of it may not be considered unlawful.

3.3.2 Front-Running
As long as front running is concerned, it is more of an unethical practice usually performed by brokers. Just like Insider-Trading, it provides the broker with very sensitive information that gives him advantage over the rest of the brokers/traders. It is defined as “when a broker or other entity enters into a trade because they have foreknowledge of a big non-publicized transaction that will influence the price of the asset, resulting in a likely financial gain for the broker” [18]. It is considered unethical and illegal. Luckily, the SEC are always regulating the market and tracking down the brokers trespassing the law.
CHAPTER IV
Empirical Results

4.1 MODEL DESIGN
In the section above, the mathematical reasoning behind the model, which has been used, has been developed. The Lagrange Multiplier Method is one of the numerous ways analysts and mathematicians use to optimize portfolios on a daily basis. The code itself is designed in a way that it would go through three different fundamental steps:

- Data Extraction “Automated from the Internet”.
- Optimization through Lagrange.
- Coding of the efficient frontier.

The language used to program the software is the R programming language. The reason why this project has opted for the use of this particular coding system is all the advantage and simplicity it offers. To be more specific, the language offers a variety of already built-in functions that do not exist in other languages. The latter facilitates the programming part of the project and makes it easier on the reader to directly understand the program implemented instead of going through the building of complex functions. Furthermore, in this project, we are not concerned with the algorithm efficiency or the time of execution, but the results of the Lagrange Multiplier method and the efficient frontier.

As stated before, this program works only with stocks. The program conducts four different simulations for different kinds of scenarios depending on the risk aversion of the potential investor. The stocks selection has been done from a list of both stock exchanges: NASDAQ & NYSE. The scenarios go from low risk, to moderate risk, to high risk. In addition to that, there has been an addition of a final scenario with its own real-life implementation on two different simulation platforms: iqoption and wallstreetsurvivor.com. Both platforms offer a big collection of stocks investors could choose from.

All the stocks chosen in the implementation of this program are active. In order to know whether they are volatile or not, the website marketwatch.com offers a selection of “most volatile” and “least volatile” stocks in the first quarter of 2019 in the American market.
Note, for every scenario, the results show the output for a required rate of return of the investor of 0.001 as an arbitrary number which can be adjusted. There are different ways of computing the required rate depending on the analyst. The most used method is the calculation of the CAPM for every single security and coming up with a weighted average CAPM for the entire portfolio. Because we do not want to confuse the reader, the program chose an arbitrary number for a clear and better understanding of the scenarios.

4.2 DIFFERENT SCENARIOS

4.2.1 – Low Risk Scenario

In the first case scenario, there has been the implementation of three non-volatile stock in the American stock Market. The reason why we have chosen to run this is due to the fact that the project takes into consideration the different types of preferences of the investor. If an investor is high risk-averse, the selection from those stocks suggested by “MarketWatch.com” could be a perfect collection of financial assets they could choose from.

![Figure 2. Least Volatile Stocks in USA, first quarter of 2019](image-url)
As it has been stated before, the portfolio should be diversified. That is why; three different companies from three different industries have been selected:

- Twenty-First Century Fox – Listed in NASDAQ with a daily price volatility as low as 0.52. **Industry**: Movies/Entertainment.
- Walmart – Listed in NASDAQ with a daily price volatility as low as 0.72. **Industry**: Special Stores.
- Western Union Co. – Listed in NYSE with a daily volatility price as low as 0.83. **Industry**: Finance/Rental/Leasing.

First, the data was collected with the help of an already built-in function “getSymbols” on R, which automatically extracts all the different kinds of prices (Open, High, Low, Closing, and Adjusted). There are different sources used by this function and the user could specify whether he/she wants his data extracted from yahoo, google, a csv file, MySQL, FRED, RData, or Oanda, but if nothing is mentioned, by default the data is extracted from yahoo finance [17]. As it has been clearly stated in the methodology part, the project works with the closing prices instead of the adjusted closing ones. The reason behind that is that there is no tangible or reliable sources stating if the companies have been offering dividends or stocks splits/offering In order to have a normal and fair valuation of all companies’ returns, the project relies on the closing price without any adjustments. After doing so, the implementation of the code (Appendix B), shows a systematic coding of what has been stated in the Methodology chapter.

As an output, the model implemented gave us the following weights:

<table>
<thead>
<tr>
<th>Stock</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOXA.Close</td>
<td>-0.1861022</td>
</tr>
<tr>
<td>WU.Close</td>
<td>-0.5244181</td>
</tr>
<tr>
<td>WMT.Close</td>
<td>1.7105203</td>
</tr>
</tbody>
</table>

Figure 3. Output of the Weights of the Low Risk Scenario

The investor should therefore, according to the model implemented, short-sell both of **Western Union Co** -52.44% and **Twenty First Century Fox stocks** -18.61%. Finally, **171.05%** in Walmart (It is over 100% because there is a borrowing of money from the two other stocks that is invested in the third one).
In the program designed, a code segment calculates the riskiness of the portfolio.

```r
#CORRESPONDING PORTFOLIO RISK USING CROSS PRODUCT OF THE WEIGHTS
risk_portfolio <- t(Weight_Vec)%*%Covariance_Matrix%*%Weight_Vec
Std_portfolio <- sqrt(risk_portfolio)
```

- In this scenario, the corresponding portfolio risk is of 0.0207 (Low).

To help the investor choose the optimal portfolio combination for those three assets, the model designed outputs the efficient frontier (explained in chapter 3). The curve plotted encourages the investor to choose the most suitable portfolio combination for him if he/she has second thoughts about the initial required return selected or there is a doubt about his risk-aversion. All the optimal solutions have been plotted in one single graph:

![Efficient Set.](image)

Figure 4. Efficient Frontier, N= three Stocks, Low-Risk Scenario.
4.2.2 – High-Risk Scenario

Just as it has been done in the first scenario, the high-risk case consists in taking three most volatile stocks of the first quarter of 2019. Once again, for the sake of evaluation the volatility of the stock, the reliance on the data provided on MarketWatch.com is primordial. The latter exhibits the daily price volatility of every stock displayed. The stocks chosen remain in both the NYSE and NASDAQ stock exchanges and from different industries.

Figure 5. Most Volatile Stocks in USA, first quarter of 2019 [16]

The diversification factor is again taken into consideration. The selection of portfolios considered for this part of the project thusly is:

- Nvidia Corp – Listed in NASDAQ with a daily price volatility as high as 4.71.
  - **Industry**: Semiconductors
- Nektar Therapeutics – Listed in NASDAQ with a daily price volatility as high as 4.54.
  - **Industry**: Biotechnology.
Freeport-McMoRan Inc. – Listed in NYSE with a *daily price volatility* as high as 4.57.

Industry: Precious Metals

Same code implementation used for the previous scenario is used in this one (*Appendix D*). The latter has given the following weights:

<table>
<thead>
<tr>
<th>Security</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>NVDA.Close</td>
<td>0.5533543</td>
</tr>
<tr>
<td>FCX.Close</td>
<td>0.9816483</td>
</tr>
<tr>
<td>NKTR.Close</td>
<td>-0.5350026</td>
</tr>
</tbody>
</table>

![Figure 6. Output of the Weights of the High Risk Scenario](image)

Here, the investor should short-sell the *Nektar Therapeutics* stocks with a weight of -53.50%, and allocate 55.34% to *Nvidia*, and 98.16% of his budget to *Freeport-McMoRan Inc*.

In this scenario, the corresponding riskiness of the portfolio is of **0.0456 (High)**.

For the efficient frontier in that case, we got the following curve:
If the curve looks linear, it may be due to the fact that we have a small number of assets in this case. Nevertheless, usually the curve is under the form of hyperbola. It will be better emphasized when we proceed to the \textit{N=10 Assets scenario}.

\textbf{4.2.3 – Moderate-Risk Scenario}

In the moderate-risk scenario, there has been a selection of three financial assets. Those sale assets were shaking the internet with news during the last quarter. Thanks to the numerous information straying on the internet on those stocks, it is only logical to invest in them since it is so simple to decide intuitively whether the stock prices are going to rise or fall. Indeed, Apple, Amazon and Tesla have all been subject to heavy attention in the media during the first quarter of 2019. Each company has introduced new products in 2019 and many positive predictions on behalf of those are to be taken into consideration. Today, thanks to social media, mainly Twitter, their CEOs express openly their minds. Analysts and investors consider those as they offer an intuition of what the company’s next move or direction may be.

The stocks chosen are:

- Apple Inc. – Listed in NASDAQ with a \textit{10-days price volatility} of 0.1380. \textbf{Industry}: Consumer Electronics

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{historical_volatility.png}
\caption{Historical Volatility of Apple Inc. Stock. (10 Day) [19]}
\end{figure}
➢ Tesla, Inc. – Listed in NASDAQ with a 10-days volatility price of 0.2742.
  ➢ Industry: Auto Manufacturers.

![Historical Volatility (Close-to-Close) (10-Day)](image)

Figure 9. Historical Volatility of Tesla Inc. Stock. (10 Day)[21]

➢ Amazon.com, Inc. – Listed in NASDAQ with a 10-days volatility price of 0.0695. ➢ Industry: Specialty Retail

![Historical Volatility (Close-to-Close) (10-Day)](image)

Figure 10. Historical Volatility of Amazon.com Inc. Stock. (10 Day)[21]

Reiteratively, the same code is implemented (Appendix C). The respective outputs of the weights for these three particular companies are:
In this case, the program suggests to the investor to short-sell only for the *Tesla* stock by -36.84%, to allocate 40.14% and 96.70% of the money respectively to *Amazon.com, Inc.*, and *Apple, Inc.*.

⇒ In this scenario, the corresponding riskiness of the portfolio is of 0.0245 (Moderate).

As for the optimal set of combinations, the following graph applies:

![Efficient Set.](image)

Figure 12. Efficient Frontier, N= three Stocks, High-Risk Scenario.
4.2.4 – A 10 Assets Case Scenario

In an attempt to *mimic* what is happening in real-life on trading floors, the model has been implemented for a portfolio that holds *10 assets* from different industries. As for as the code is concerned, the time of execution would be longer since there is more data to take into consideration. The code implemented is displayed on Appendix F.

The stocks chosen are summarized in the following table:

<table>
<thead>
<tr>
<th>Stock</th>
<th>Industry</th>
<th>Listing</th>
<th>10-days volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intel Corporation</td>
<td>Semiconductors</td>
<td>NASDAQ</td>
<td>0.1873</td>
</tr>
<tr>
<td>eBay Inc.</td>
<td>Specialty Retail</td>
<td>NASDAQ</td>
<td>0.2315</td>
</tr>
<tr>
<td>Microsoft</td>
<td>Software- Infrastructure</td>
<td>NASDAQ</td>
<td>0.0839</td>
</tr>
<tr>
<td>Facebook</td>
<td>Internet Content &amp; Information</td>
<td>NASDAQ</td>
<td>0.0706</td>
</tr>
<tr>
<td>Diamond Offshore</td>
<td>Oil &amp; Gas Drilling</td>
<td>NYSE</td>
<td>0.1741</td>
</tr>
<tr>
<td>3D Systems Corp</td>
<td>Computer Systems</td>
<td>NYSE</td>
<td>0.1434</td>
</tr>
<tr>
<td>3M Company</td>
<td>Diversified Industrials</td>
<td>NYSE</td>
<td>0.1205</td>
</tr>
<tr>
<td>Tesla, Inc.</td>
<td>Auto Manufacturers</td>
<td>NASDAQ</td>
<td>0.2742</td>
</tr>
<tr>
<td>Netflix, Inc.</td>
<td>Media Diversified</td>
<td>NASDAQ</td>
<td>0.3865</td>
</tr>
<tr>
<td>Ali Baba</td>
<td>Specialty Retail</td>
<td>NYSE</td>
<td>0.2761</td>
</tr>
</tbody>
</table>

Figure 13. Selection of 10 Listed Diversified Active Companies [22],[23].

The program implemented outputs the following weights:

Figure 14. Output of the Weights for the 10 Assets Scenario
In this scenario, the corresponding riskiness of the portfolio is of 0.01479358 (Low, thanks to diversification and the number of assets).

The corresponding efficient frontier for the above selection is represented with the following curve:

![Efficient Frontier for the 10 Assets Scenario](image)

**Figure 15. Efficient Frontier for the 10 Assets Scenario**

### 4.2.5 – Random-Case Scenario with Real-life simulation

#### 4.2.5.1 – Model Outputs

To compare the results of the model implemented with what would actually happen in real life, another simulation has been performed. The reason why no previous simulation has been taken and compared is due to the short-selling factor. Some platforms online, like iqoption, do not allow the short-selling factor.

The following combination of stocks has allowed a strictly positive weight distribution between the assets: Apple Inc., Amazon.com, Inc., and Advanced Micro Devices, Inc.
The latter has given the following set of weights:

<table>
<thead>
<tr>
<th>Stock</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAPL.Close</td>
<td>0.69346024</td>
</tr>
<tr>
<td>AMD.Close</td>
<td>0.24296454</td>
</tr>
<tr>
<td>AMZN.Close</td>
<td>0.06357523</td>
</tr>
</tbody>
</table>

Figure 16. Optimal Weights, short selling excluded

In this scenario, the corresponding riskiness of the portfolio is 0.02081585.

Like the previous scenarios, the required rate of return used is 0.001.

The efficient frontier for this simulation looks like the following:

Figure 17. Efficient-Frontier, last simulation (Short-Selling Excluded).
In the model simulation, the required rate of the investor was of 0.001 (0.01%) for an investment of around 1000$, not taking into consideration the transaction fees. In this case, the portfolio is doing even better than what was expected with a rate of return of +1.85%.

The percentage difference between what was required and what actually is happening is 18.48%. It has been calculated as follows:

- Actual Return – Expected Required Return
- Actual Return

Figure 18. Real Life Simulation with IQ OPTION.

The simulation on Wallstreetsurvivor.com gave around the same results.

**Discussion and Analysis**

There could be many explanations as to why there is a difference between what was simulated and what actually occurred. Although the difference between the two results is not shocking, it is important to keep in mind that the model implementation takes into account many assumptions. Some of them are listed in the first part of this project (Markowitz Portfolio Theory Assumptions). Another thing is that this program takes for granted that the covariance between the companies chosen is always fixed. In real life, this is no true. For this reason, a program that constantly updates the information on what is happening to the market is essential.
4.3 TIME COMPLEXITY OF THE MODEL DESIGNED

The challenges that analysts find in real life as far as portfolio optimization is concerned are numerous. One of them is the time complexity of the program that they are working with. Indeed, usually portfolios contain a big number of assets. Thus, the data is big, and the implementation requires sometimes that works fast for an immediate simulation and output.

Before talking about the time complexity of a program, it is important to define it. The time complexity of a program usually refers to the running time. Because it is very difficult to know a code’s precise operation time since it depends on several factors hard to catch, for instance the processor’s speed, it is common to estimate the order of magnitude “Big O Notations”. [24]

There are several comments on the model implemented stating every function’s big O notation to perform an estimation of the actual time complexity (Appendix E). The biggest O notation of the program remain in the first bit of it, where we try to calculate the returns. Since we are working with around 250 data per asset, it is \( O(250*N) \).

If the program could be implemented using a different programming language, such as Python or Java since the running time would be smaller. Most importantly, it would depend on how the Algorithm has been implemented. If most of the functions were linear then the model would output very fast.
Chapter V

RECOMMENDATIONS, FUTURE WORK

Briefly, in order to go further within the realm of this project, it would be recommended to implement the program in another language with a higher time complexity. In addition to that, the model would be much more efficient if the data extracted was constantly updated. Accounting for the variation of the covariance between stocks is highly recommended because it would be more realistic and less limiting. Besides, the computation of the expected required rate of return of the investor is usually done using the CAPM model instead of just an arbitrary number since it takes into consideration the risk free as well as the market risk premium of the market.
CONCLUSION

The model designed is a very fundamental tool that any investor can use as a starting point in the trading industry. It gives an insight to how a portfolio could be implemented and what could be its limitations and strength.

The algorithm offers a very simplified view on how a model as complex as the Lagrangian Multipliers could be coded in a very easy and simple way. In addition to that, it clearly exhibits the effect of diversifications. As a starter in the trading world, the program offers a reliable selection of portfolio weights. In addition to that, the efficient frontier is essential in the case of a change of heart from the investor regarding his risk aversion.

The time complexity of the program is significant, but even that could be optimized. Nonetheless, thanks to the built-in functions already offered in R, the investor understands without any complications the purpose of every single line of code.

Despite the different limitations of the program, it still offers a fair simulation, which could still be used in real life situations.
REFERENCES


APPENDIX A – Project Specifications

ASMAMA Oumaima
EMS
INVESTMENT STRATEGIES: PORTFOLIO & RISK OPTIMIZATION WITH R.
LAAYOUNI L
Spring 2019

The aim behind this capstone project is to come up with an optimal strategy to help investors maximize their wealth while minimizing the risk taken when investing in financial instruments, portfolios, using statistical methods with the help of the R programming language, and linear algebra.

Implementing this project will require the need of some analysis. First, there will be an analysis regarding the different factors that must be taken into consideration when investing such as: risk evaluation, expected return, and variance. Next, there will be an identification of the different tendencies of a potential investor in a market as hectic as the NYSE (New York Stock Exchange), and analyse the kind of efficient sets a typical investor would go for in a heavily volatile market using historical data. The following step will consist of optimizing the portfolio either by setting a fixed profit with the minimum risk possible, or maximizing the gains for a given tolerance of risk.

After doing so, the next step will consist of finding the most optimal weights and performing a scenario analysis; this will lead us to choosing some cases of different asset combinations, and come up with a certain optimization model, which will go along with the risk aversion of a potential X or Y investor. The objective is to optimize the portfolio in the sense that it will have a high-expected return and a low standard deviation of return. Following this step, the results will be simplified so that any investor with no statistical or finance background will be able to get a hand over the different concepts presented and use them to his/her advantage.

The statistical approach, which will consist of coding in R, will enable any investor with a certain amount of risk tolerance or target profit to have the script output the weights needed for every financial security data he/she provided the coding program with. The investor will need to know the number of securities he/she wants to invest in and the rate of returns.

As for the societal implications of this project, the methods presented aim to optimize portfolio combinations for investors. It is meant to assist them with their investment decisions. Like any finance model the one presented will most likely not be considered as unequivocal but will certainly provide a certain assistance and guidance depending on the risk aversion of the investors themselves.

Regarding the ethical implications, because investment decisions imply a certain consent from the investor to take a certain amount of risk, the model and implementation provided for every portfolio combination taken will state the degree of risk aversion required. Thus, the investor will be fully aware of the amount of risk taken and of the model’s accuracy. In addition to that, any information used will be attributed to its author.
# Installing Packages Needed for this simulation.
install.packages('quantmod')
install.packages('fPortfolio')

# Installing libraries needed for this simulation.
library('quantmod')

# IMPORTANT VARIABLES
N = 3 # Code Simulation for N = 3 Assets - Financial Stocks in the American Market
Maximum_return = 0.175 # Maximum Return Considered
P_NUMB=150 # Number of Portfolios Considered

# DATA EXTRACTION
getSymbols('FOXA', from = "2018-03-26")
getSymbols('WMT', from = "2018-03-26")
getSymbols('WU', from = "2018-03-26")

# Get the Closed Prices to estimate return
Twenty_First_Cent <- Cl(FOXA)
Walmart <- Cl(WMT)
Western_Union <- Cl(WU)

# Ploting historical prices for accurate visualization
plot(FOXA[, "FOXA.Closed"], main = "FOXA")
candleChart(FOXA, up.col = "green", dn.col = "red", theme = "white")

plot(WMT[, "WMT.Closed"], main = "WMT")
candleChart(WMT, up.col = "green", dn.col = "red", theme = "white")

plot(WU[, "WU.Closed"], main = "WU")
candleChart(WU, up.col = "green", dn.col = "red", theme = "white")

# Binding All the stocks into one single Matrix
Stocks1 <- cbind(Twenty_First_Cent, Walmart, Western_Union)

# Historical daily return
Twenty_First_Cent_d <- diff(Twenty_First_Cent) # this calculates the differences
Twenty_First_Cent_d <- Twenty_First_Cent_d[2:length(Twenty_First_Cent_d)] # drop the 1st observation from the first differences
Twenty_First_Cent_ror <- Twenty_First_Cent_d / Twenty_First_Cent # calculates the ROR

Walmart_d <- diff(Walmart) # this calculates the differences
Walmart_d <- Walmart_d[2:length(Walmart_d)] # drop the 1st observation from the first differences
Walmart_ror <- Walmart_d / Walmart # calculates the ROR

Western_Union_d <- diff(Western_Union) # this calculates the differences
Western_Union_d <- Western_Union_d[2:length(Western_Union_d)] # drop the 1st observation from the first differences
Western_Union_ror <- Western_Union_d / Western_Union # calculates the ROR
# Creating the ROR Matrix

```r
RoR_Matrix <- cbind (Twenty_First_Cent_ror, Western_Union_ror, Walmart_ror)
```

# Computing the Arithmetic mean of the daily rate of returns of the Assets chosen:

```r
Asset_mean <- colMeans(RoR_Matrix) # Calculating the mean for each Company's daily rate of returns
```

# Covariance: The covariance is an important element in the Method used to optimize the portfolio.

Notice: In Lagrange Method, the covariance since: Cov between 2 elements is simply...

...the correlation between the two time the standard deviation of each.

```r
Covariance_Matrix <- cov(RoR_Matrix, RoR_Matrix)
```

# Lagrangian Method; used for the optimization (minimizing the risk) for a !!!!!!!GIVEN REQUIRED RATE OF RETURN!!!!!!.

# Further explanation about the choice of the required rate of return is provided in the report.

```r
Investor_rrr = 0.001
```

# Finding Cov^-1 or the Inverse of the Covariance.

```r
Inverse_Covariance <- solve(Covariance_Matrix)
```

# Initially create an Empty Matrix A of 2 rows and 2 columns

```r
Z = matrix(c(rep.int(0,4)), nrow=2)
```

# Constraint consisting in the fact that the sum of all the weights equates 1; We have as many 1s as we have the number of stocks

```r
Const1=c(rep.int(1, N))
```

# Using the Lagrange Method to Find the Weights

```r
s11 <- Z[1,1] <- Const1%*%Inverse_Covariance%*%Const1
Z[2,1] <- Asset_mean%*%Inverse_Covariance%*%Const1
s21 <- Z[1,2]
s12 <- s21
s22 <- Z[2,2] <- Asset_mean%*%Inverse_Covariance%*%Asset_mean
NewVec = c(2,2*Investor_rrr)
```

# Finding the Lagrangians

```r
uc <- solve(Z, NewVec)
```

# Now, the weights are the only left unknown variables.

# Calculating, we get the following:

```r
Weight_Vec <- Inverse_Covariance%*%(u[1]*Const1+u[2]*Asset_mean)/2
```

# Printing the Optimal Weights obtained from the Lagrange Multipliers Method Calculated above

```r
Weight_Vec
```

# CORRESPONDING PORTFOLIO RISK USING CROSS PRODUCT OF THE WEIGHTS

```r
risk_portfolio <- t(Weight_Vec)%*%Covariance_Matrix%*%Weight_Vec
Std_portfolio <- sqrt(risk_portfolio)
```

# BUILDING THE EFFICIENT FRONTIER

```r
Exp_Return_Portfolio = c(rep.int(0, P_NUMB))
Std_Dev_Portfolio = c(rep.int(0, P_NUMB))
```
x<-matrix(c(rep.int(0,3*P_NUMB)),nrow=3)
y<-matrix(c(rep.int(0,2*P_NUMB)),nrow=2)
z<-matrix(c(rep.int(0,2*P_NUMB)),nrow=2)

#Creating a for loop to avoid redundancy
for(j in 1:P_NUMB)
{
    Exp_Return_Portfolio[j]<-j*Maximum_return/P_NUMB
    z[,j]<-c(2,2*Exp_Return_Portfolio[j])
    y[,j]<-solve(Z,z[,j])
    x[,j]<-Inverse_Covariance%*%(y[1,j]*Const1+y[2,j]*Asset_mean)/2

    #Standard dev for every j
    Std_Dev_Portfolio[j] <- sqrt(t(x[,j]) %*% Covariance_Matrix %*% x[,j])
}

#Finally, plotting the efficient Frontier.
plot(Std_Dev_Portfolio,Exp_Return_Portfolio, main ="Efficient Set.",col= "black", pch = 0)
APPENDIX C – R Code Implementation for Moderate-Risk Scenario.

```r
# Installing Packages Needed for this simulation.
install.packages('quantmod')
install.packages('fPortfolio')

# Installing libraries needed for this simulation.
library('quantmod')

# IMPORTANT VARIABLES
N = 3  # Code Simulation for N = 3 Assets - Financial Stocks in the American Market
Max_return = 0.05  # Maximum Return Considered
P_NUMB=100  # Number of Portfolios Considered

# DATA EXTRACTION
getSymbols('AAPL', from = "2018-03-26")
getSymbols('AMZN', from = "2018-03-26")
getSymbols('TSLA', from = "2018-03-26")

# Get the Adjusted Prices to estimate return
Apple <- Cl(AAPL)
Amazon <- Cl(AMZN)
Tesla <- Cl(TSLA)

# Plotting historical prices for accurate visualization
plot(AAPL[, "AAPL.Adjusted"], main = "AAPL")
candleChart(AAPL, up.col = "green", dn.col = "red", theme = "white")

plot(AMZN[, "AMZN.Adjusted"], main = "AMZN")
candleChart(AMZN, up.col = "green", dn.col = "red", theme = "white")

plot(TSLA[, "TSLA.Adjusted"], main = "TSLA")
candleChart(TSLA, up.col = "green", dn.col = "red", theme = "white")

# Binding All the stocks into one single Matrix
Stocks1 <- cbind(Apple, Amazon, Tesla)

# Historical daily return
apple_d <- diff(Apple)  # this calculates the differences
apple_d <- apple_d[2:length(apple_d)]  # drop the 1st observation from the first differences
apple_ror <- apple_d / Apple  # calculates the ROR

amazon_d <- diff(Amazon)  # this calculates the differences
amazon_d <- amazon_d[2:length(amazon_d)]  # drop the 1st observation from the first differences
amazon_ror <- amazon_d / Amazon  # calculates the ROR

tesla_d <- diff(Tesla)  # this calculates the differences
tesla_d <- tesla_d[2:length(tesla_d)]  # drop the 1st observation from the first differences
tesla_ror <- tesla_d / Tesla  # calculates the ROR

# Creating the ROR Matrix
```

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RoR_Matrix <- cbind(apple_ror, tesla_ror, amazon_ror)

# Computing the Arithmetic mean of the daily rate of returns of the Assets chosen:
Asset_mean <- colMeans(RoR_Matrix)  # Calculating the mean for each Company's daily rate of returns

# Covariance: The covariance is an important element in the Method used to optimize the portfolio.
# Notice: In Lagrange Method, the covariance since: Cov between 2 elements is simply...
# ...the correlation between the two time the standard deviation of each.
Covariance_Matrix <- cov(RoR_Matrix, RoR_Matrix)

# Lagrangian Method; used for the optimization (minimizing the risk) for a !!!!!!!GIVEN REQUIRED RATE OF RETURN!!!!!!.
# Further explanation about the choice of the required rate of return is provided in the report.
Investor_rrr = 0.001

# Creation of an empty vector
Weight_Vec <- c(rep.int(0, N))

# Finding Cov^-1 or the Inverse of the Covariance.
Inverse_Covariance <- solve(Covariance_Matrix)

# Initially create an Empty Matrix A of 2 rows and 2 columns
Z = matrix(c(rep.int(0,4)), nrow=2)

# Constraint consisting in the fact that the sum of all the weights equates 1; We have as many 1s as we have the number of stocks
Const1 = c(rep.int(1, N))

# Using the Lagrange Method to Find the Weights
s11 <- Z[1,1] <- Const1**Inverse_Covariance**Const1
Z[2,1] <- Asset_mean**Inverse_Covariance**Const1
Z[1,2] <- Z[2,1]
s21 <- Z[1,2]
s12 <- s21
s22 <- Z[2,2] <- Asset_mean**Inverse_Covariance**Asset_mean
NewVec = c(2, 2*Investor_rrr)

# Finding the Lagrangians
u <- solve(Z, NewVec)

# Now, the weights are the only left unknown variables.
# Calculating, we get the following:
Weight_Vec <- Inverse_Covariance**u[1]**Const1 + u[2]**Asset_mean)/2

# Printing the Optimal Weights obtained from the Lagrange Multipliers Method Calculated above
Weight_Vec

# CORRESPONDING PORTFOLIO RISK USING CROSS PRODUCT OF THE WEIGHTS
risk_portfolio <- t(Weight_Vec)**Covariance_Matrix**Weight_Vec
Std_portfolio <- sqrt(risk_portfolio)

# BUILDING THE EFFICIENT FRONTIER
Exp_Return_Portfolio = c(rep.int(0, P_NUMB))
\[
\text{Std}_\text{Dev}_\text{Portfolio} = c(\text{rep.int}(0, P\_NUMB))
\]

\[
x <- \text{matrix}(c(\text{rep.int}(0, 3^*P\_NUMB)), nrow=3)
y <- \text{matrix}(c(\text{rep.int}(0, 2^*P\_NUMB)), nrow=2)
z <- \text{matrix}(c(\text{rep.int}(0, 2^*P\_NUMB)), nrow=2)
\]

# Creating a for loop to avoid redundancy
for (j in 1:P\_NUMB)
{
    Exp\_Return\_Portfolio[j] <- j*Maximum\_return/P\_NUMB
    z[,j] <- c(2, 2*Exp\_Return\_Portfolio[j])
y[,j] <- solve(Z, z[,j])
x[,j] <- Inverse\_Covariance\%\%y[1, j]*Const1+y[2, j]*Asset\_mean)/2
    # Standard dev for every j
    Std\_Dev\_Portfolio[j] <- sqrt(t(x[,j]) \%\% Covariance\_Matrix \%\% x[,j])
}

# Finally, plotting the efficient Frontier.
plot(Std\_Dev\_Portfolio, Exp\_Return\_Portfolio, main ="Efficient Set.", col = "black", pch = 0)
APPENDIX D – R Code Implementation for High-Risk Scenario.

# Installing Packages Needed for this simulation.
install.packages('quantmod')
install.packages('fPortfolio')

# Installing libraries needed for this simulation.
library('quantmod')

# IMPORTANT VARIABLES
N = 3 # Code Simulation for N = 3 Assets - Financial Stocks in the American Market
Maximum_return = 0.05 # Maximum Return Considered
P_NUMB = 100 # Number of Portfolios Considered

# DATA EXTRACTION
getSymbols('NVDA', from = "2018-03-26")
getSymbols('DVN', from = "2018-03-26")
getSymbols('FCX', from = "2018-03-26")

# Get the Adjusted Prices to estimate return
NVIDIA <- Cl(NVDA)
DVON_ENERGY <- Cl(DVN)
FREEPORT_MCMORAN <- Cl(FCX)

# Plotting historical prices for accurate visualization
plot(NVDA[, "NVDA.Adjusted"], main = "NVDA")
candleChart(NVDA, up.col = "green", dn.col = "red", theme = "white")

plot(DVN[, "DVN.Adjusted"], main = "DVN")
candleChart(DVN, up.col = "green", dn.col = "red", theme = "white")

plot(FCX[, "FCX.Adjusted"], main = "FCX")
candleChart(FCX, up.col = "green", dn.col = "red", theme = "white")

# Binding All the stocks into one single Matrix
Stocks1 <- cbind(NVIDIA, DVON_ENERGY, FREEPORT_MCMORAN)

# Historical daily return
NVIDIA_d <- diff(NVIDIA) # this calculates the differences
NVIDIA_d <- NVIDIA_d[2:length(NVIDIA_d)] # drop the 1st observation from the first differences
NVIDIA_ror <- NVIDIA_d / NVIDIA # calculates the ROR

DVON_ENERGY_d <- diff(DVON_ENERGY) # this calculates the differences
DVON_ENERGY_d <- DVON_ENERGY_d[2:length(DVON_ENERGY_d)] # drop the 1st observation from the first differences
DVON_ENERGY_ror <- DVON_ENERGY_d / DVON_ENERGY # calculates the ROR

FREEPORT_MCMORAN_d <- diff(FREEPORT_MCMORAN) # this calculates the differences
FREEPORT_MCMORAN_d <- FREEPORT_MCMORAN_d[2:length(FREEPORT_MCMORAN_d)] # drop the 1st observation from the first differences
FREEPORT_MCMORAN_ror <- FREEPORT_MCMORAN_d / FREEPORT_MCMORAN # calculates the ROR

# Creating the ROR Matrix
RoR_Matrix <- cbind(NVIDIA_ror, FREEPORT_MCMORAN_ror, DVON_ENERGY_ror)
#Computing the Arithmetic mean of the daily rate of returns of the Assets chosen:
Asset_mean <- colMeans(RoR_Matrix)  #Calculating the mean for each Company's daily rate of returns

#Covariance: The covariance is an important element in the Method used to optimize the portfolio.
#Notice: In Lagrange Method, the covariance since: Cov between 2 elements is simply...
# the correlation between the two time the standard deviation of each.
Covariance_Matrix <- cov(RoR_Matrix, RoR_Matrix)

#Lagrangian Method; used for the optimization (minimizing the risk) for a !!!!!GIVEN REQUIRED RATE OF RETURN!!!!!!.
#Further explanation about the choice of the required rate of return is provided in the rep.intort.
Investor_rrr = 0.001

#Creation of an empty vector
Weight_Vec <- c(rep.int(0, N))

#Finding Cov^-1 or the Inverse of the Covariance.
Inverse_Covariance <- solve(Covariance_Matrix)

#Initially create an Empty Matrix A of 2 rows and 2 columns
Z = matrix(c(rep.int(0, 4)), nrow=2)

# Constraint consisting in the fact that the sum of all the weights equates 1; We have as many 1s as we have the number of stocks
Const1 = c(rep.int(1, N))

#Using the Lagrange Method to Find the Weights
s11 <- Z[1,1] <- Const1*Inverse_Covariance*Const1
Z[1,2] <- Z[2,1]
s21 <- Z[1,2]
s12 <- s21
s22 <- Z[2,2] <- Asset_mean*Inverse_Covariance*Asset_mean
NewVec = c(2, 2*Investor_rrr)

#Finding the Lagrangians
uc <- solve(Z, NewVec)

#Now, the weights are the only left unknown variables.
#Calculating, we get the following:
Weight_Vec <- Inverse_Covariance*(u[1]*Const1 + u[2]*Asset_mean)/2

#Printing the Optimal Weights obtained from the Lagrange Multipliers Method Calculated above
Weight_Vec

#CORRESPONDING PORTFOLIO RISK USING CROSS PRODUCT OF THE WEIGHTS
risk_portfolio <- t(Weight_Vec)*Covariance_Matrix*Weight_Vec
Std_portfolio <- sqrt(risk_portfolio)

#BUILDING THE EFFICIENT FRONTIER

Exp_Return_Portfolio = c(rep.int(0, N))
Std_Dev_Portfolio = c(rep.int(0, N))
#Creating a for loop to avoid redundance
for(j in 1:P_NUMB) {
  Exp_Return_Portfolio[j] <- j*Maximum_return/P_NUMB
  z[,j] <- c(2,2*Exp_Return_Portfolio[j])
  y[,j] <- solve(Z,z[,j])
  x[,j] <- Inverse_Covariance%*%(y[1,j]*Const1+y[2,j]*Asset_mean)/2

  #Standard dev for every j
  Std_Dev_Portfolio[j] <- sqrt(t(x[,j]) %*% Covariance_Matrix %*% x[,j])
}

#Finally, plotting the efficient Frontier.
plot(Std_Dev_Portfolio,Exp_Return_Portfolio, main = "Efficient Set.",col = "black", pch = 0)
#Installing Packages Needed for this simulation.
install.packages('quantmod')
install.packages('fPortfolio')

#Installing libraries needed for this simulation.
library('quantmod')

#IMPORTANT VARIABLES
N = 3 # Code Simulation for N = 3 Assets -Financial Stocks in the American Market
Maximum_return = 0.04 #Maximum Return Considered
P_NUMB=75 #Number of Portfolios Considered

#DATA EXTRACTION
getSymbols('AAPL', from = "2018-03-26")
getSymbols('AMZN', from = "2018-03-26")
getSymbols('AMD', from = "2018-03-26")

#Get the Adjusted Prices to estimate return
Apple <- Cl(AAPL)
Amazon <- Cl(AMZN)
X <- Cl(AMD)

#Binding All the stocks into one single Matrix
Stocks1 <- cbind(Apple, Amazon, X)

#Historical daily return
apple_d <- diff(Apple) #this calculates the differences O(n) with n being the size of the column or array
apple_d <- apple_d[2:length(apple_d)] #drop the 1st observation from the first differences O(1) constant time
apple_ror <- apple_d / Apple #calculates the ROR O(n) linear time

amazon_d <- diff(Amazon) #this calculates the differences
amazon_d <- amazon_d[2:length(amazon_d)] #drop the 1st observation from the first differences
amazon_ror <- amazon_d / Amazon #calculates the ROR

x_d <- diff(X) #this calculates the differences
x_d <- x_d[2:length(x_d)] #drop the 1st observation from the first differences
x_ror <- x_d / X #calculates the ROR

#Creating the ROR Matrix
RoR_Matrix <- cbind(apple_ror, x_ror, amazon_ror)

#Computing the Arithmetic mean of the daily rate of returns of the Assets chosen:
Asset_mean <- colMeans(RoR_Matrix) #Calculating the mean for each Company's daily rate of returns O(n) for every column so actually O(3*n) = O(n) (m = 3, number of stocks/features)

#Covariance : The covariance is an important element in the Method used to optimize the portfolio.
In Lagrange Method, the covariance since: Cov between 2 elements is simply the correlation between the two time the standard deviation of each.

\[
\text{Covariance Matrix} \leftarrow \text{cov}(\text{RoR Matrix}, \text{RoR Matrix}) \quad \mathcal{O}(n^2m^2) \text{ with } m \text{ being the number of stocks. For } m = 3, \mathcal{O}(9n) = \mathcal{O}(n) \text{ (Small constant factor. Can be ignored)}
\]

Lagrangian Method; used for the optimization (minimizing the risk) for a GIVEN REQUIRED RATE OF RETURN!!!!!!.

Further explanation about the choice of the required rate of return is provided in the report.

\[
\text{Investor}_r = 0.001
\]

Creation of an empty vector

\[
\text{Weight Vec} \leftarrow \text{c(rep.int}(0,N)) \quad \mathcal{O}(N)
\]

Finding Cov^-1 or the Inverse of the Covariance.

\[
\text{Inverse Covariance} \leftarrow \text{solve}(\text{Covariance Matrix}) \quad \mathcal{O}(m^3) \text{ with } m = 3, \text{ number of stocks/features. Gaussian Elimination normally takes cubic time}
\]

Initially create an Empty Matrix A of 2 rows and 2 columns

\[
\text{Z} = \text{matrix}(c(\text{rep.int}(0,4)), \text{nrow}=2) \quad \mathcal{O}(1)
\]

Constraint consisting in the fact that the sum of all the weights equates 1; We have as many 1s as we have the number of stocks

\[
\text{Const1} = \text{c(rep.int}(1,N)) \quad \mathcal{O}(N)
\]

Using the Lagrange Method to Find the Weights

\[
\text{s11} \leftarrow \text{Z}[1,1] \leftarrow \text{Const1} \times \text{Inverse Covariance} \times \text{Const1} \quad \text{Matrix multiplication} = \mathcal{O}(N^3) \text{ where } N \text{ is the largest dimension in a matrix}
\]

\[
\text{Z}[2,1] \leftarrow \text{Asset mean} \times \text{Inverse Covariance} \times \text{Const1}
\]

\[
\text{Z}[1,2] \leftarrow \text{Z}[2,1]
\]

\[
\text{s11} \leftarrow \text{Z}[1,1]
\]

\[
\text{s12} \leftarrow \text{Z}[1,2]
\]

\[
\text{s22} \leftarrow \text{Z}[2,2] \leftarrow \text{Asset mean} \times \text{Inverse Covariance} \times \text{Asset mean}
\]

\[
\text{NewVec} = \text{c}(2,2 \times \text{Investor}_r)
\]

Finding the Lagrangians

\[
\text{u} \leftarrow \text{solve}(\text{Z}, \text{NewVec})
\]

Now, the weights are the only left unknown variables.

Calculating, we get the following:

\[
\text{Weight Vec} \leftarrow \text{Inverse Covariance} \times \text{(u[1] \times Const1 + u[2] \times Asset mean)} / 2
\]

Printing the Optimal Weights obtained from the Lagrange Multipliers Method Calculated above

\[
\text{Weight Vec}
\]

CORRESPONDING PORTFOLIO RISK USING CROSS PRODUCT OF THE WEIGHTS

\[
\text{risk portfolio} \leftarrow \text{t(Weight Vec)} \times \text{Covariance Matrix} \times \text{Weight Vec}
\]

\[
\text{Std portfolio} \leftarrow \text{sqrt(risk portfolio)} \quad \mathcal{O}(C \times N^2) \text{ where } C \text{ is a constant and } N \text{ is the of the matrix}
\]

BUILDING THE EFFICIENT FRONTIER

\[
\text{Exp Return Portfolio} = \text{c(rep.int}(0,\text{P_NUMB}))
\]

\[
\text{Std Dev Portfolio} = \text{c(rep.int}(0,\text{P_NUMB}))
\]

\[
\text{x} \leftarrow \text{matrix}(\text{c(rep.int}(0,3 \times \text{P_NUMB})), \text{nrow}=3)
\]
```r
y <- matrix(c(rep.int(0, 2 * P_NUMB)), nrow = 2)
z <- matrix(c(rep.int(0, 2 * P_NUMB)), nrow = 2)

# Creating a for loop to avoid redundancy O(P_num * N^3)
for (j in 1:P_NUMB) {
    Exp_Return_Portfolio[j] <- j * Maximum_return / P_NUMB
    z[, j] <- c(2, 2 * Exp_Return_Portfolio[j])
    y[, j] <- solve(Z, z[, j])
    x[, j] <- Inverse_Covariance %*% (y[1, j] * Const1 + y[2, j] * Asset_mean) / 2

    # Standard dev for every j
    Std_Dev_Portfolio[j] <- sqrt(t(x[, j]) %*% Covariance_Matrix %*% x[, j])
}

# Finally, plotting the efficient Frontier.
plot(Std_Dev_Portfolio, Exp_Return_Portfolio, main = "Efficient Set.", col = "black")
```

#Installing Packages Needed for this simulation.
install.packages('quantmod')
install.packages('FPortfolio')

#Installing libraries needed for this simulation.
library('quantmod')

#IMPORTANT VARIABLES
N = 10 # Code Simulation for N = 10 Assets - Financial Stocks in the American Market
Maximum_return = 0.005 # Maximum Return Considered
P_NUMB=100 # Number of Portfolios Considered

#DATA EXTRACTION
getSymbols('INTC', from = "2018-03-26") # INTEL
getSymbols('EBAY', from = "2018-03-26") # eBay
getSymbols('MSFT', from = "2018-03-26") # Microsoft
getSymbols('FB', from = "2018-03-26") # Facebook
getSymbols('DO', from = "2018-03-26") # Diamond Offshore
getSymbols('DDD', from = "2018-03-26") # 3D SYSTEMS CORPORATION
getSymbols('MMM', from = "2018-03-26") # 3M
getSymbols('TSLA', from = "2018-03-26") # Tesla
getSymbols('NFLX', from = "2018-03-26") # Netflix
getSymbols('BABA', from = "2018-03-26") # Alibaba

#Get the adjusted Prices to estimate return
intel <- Cl(INTC) #1
ebayy <- Cl(EBAY) #2
microsoft <- Cl(MSFT)#3
facebook <- Cl(FB)#4
diamondoffshore <- Cl(DO)#5
threedsystems <- Cl(DDD)#6
threem <- Cl(MMM)#7
tesla <- Cl(TSLA)#8
netflix <- Cl(NFLX)#9
alibaba <- Cl(BABA)#10

#Binding All the stocks into one single Matrix
Stocks1 <- cbind(intel, ebayy, microsoft, facebook, diamondoffshore, threedsystems, threem, tesla, netflix, alibaba)

#Historical daily return
#1
intel_d <- diff (intel) # this calculates the differences
intel_ror <- intel_d[2:length(intel_d)] # drop the 1st observation from the first differences
#2
ebayy_d <- diff(ebayy) # this calculates the differences
ebayy_ror <- ebayy_d[2:length(ebayy_d)] # drop the 1st observation from the first differences
#3
microsoft_d <- diff(microsoft) # this calculates the differences
microsoft_ror <- microsoft_d[2:length(microsoft_d)] # drop the 1st observation from the first differences
#4
facebook_d <- diff(facebook) #this calculates the differences
facebook_d <- facebook_d[2:length(facebook_d)] #drop the 1st observation from the first differences
facebook_ror <- facebook_d / facebook #calculates the ROR

#5
diamondoffshore_d <- diff(diamondoffshore) #this calculates the differences
diamondoffshore_d <- diamondoffshore_d[2:length(diamondoffshore_d)] #drop the 1st observation from the first differences
diamondoffshore_ror <- diamondoffshore_d / diamondoffshore #calculates the ROR

#6
threedsystems_d <- diff(threedsystems) #this calculates the differences
threedsystems_d <- threedsystems_d[2:length(threedsystems_d)] #drop the 1st observation from the first differences
threedsystems_ror <- threedsystems_d / threedsystems #calculates the ROR

#7
threem_d <- diff(threem) #this calculates the differences
threem_d <- threem_d[2:length(threem_d)] #drop the 1st observation from the first differences
threem_ror <- threem_d / threem #calculates the ROR

#8
tesla_d <- diff(tesla) #this calculates the differences
tesla_d <- tesla_d[2:length(tesla_d)] #drop the 1st observation from the first differences
tesla_ror <- tesla_d / tesla #calculates the ROR

#9
netflix_d <- diff(netflix) #this calculates the differences
netflix_d <- netflix_d[2:length(netflix_d)] #drop the 1st observation from the first differences
netflix_ror <- netflix_d / netflix #calculates the ROR

#10
alibaba_d <- diff(alibaba) #this calculates the differences
alibaba_d <- alibaba_d[2:length(alibaba_d)] #drop the 1st observation from the first differences
alibaba_ror <- alibaba_d / alibaba #calculates the ROR

#Rate of Return Matrix
RoR_Matrix <- cbind(intel_ror, ebay_ror, microsoft_ror, facebook_ror, diamondoffshore_ror, threedsystems_ror, threem_ror, tesla_ror, netflix_ror, alibaba_ror)

#Computing the Arithmetic mean of the daily rate of returns of the Assets chosen:
Asset_mean <- colMeans(RoR_Matrix) #Calculating the mean for each Company's daily rate of returns

#Covariance : The covariance is an important element in the Method used to optimize the portfolio.
#Notice: In Lagrange Method, the covariance since: Cov between 2 elements is simply the correlation between the two time the standard deviation of each.
Covariance_Matrix <- cov(RoR_Matrix, RoR_Matrix)

#Lagrangian Method; used for the optimization (minimizing the risk) for a ?????GIVEN REQUIRED RATE OF RETURN!!!!!!.
#Further explanation about the choice of the required rate of return is provided in the report.
Investor_rrr = 0.001

#Creation of an empty vector
Weight_Vec <- c(rep.int(0, N))
#Finding Cov^{-1} or the Inverse of the Covariance.

\[ \text{Inverse Covariance} \leftarrow \text{solve(Covariance Matrix)} \]

#Initially create an Empty Matrix A of 2 rows and 2 columns

\[ Z = \text{matrix(c(rep.int(0, 4)), nrow=2)} \]

# Constraint consisting in the fact that the sum of all the weights equates 1; We have as many 1s as we have the number of stocks

\[ \text{Const1} = \text{c(rep.int(1, N))} \]

#Using the Lagrange Method to Find the Weights

\[ s11 <- Z[1,1] \leftarrow \text{Const1} \%*\% \text{Inverse Covariance} \%*\% \text{Const1} \]

\[ Z[2,1] <- \text{Asset mean} \%*\% \text{Inverse Covariance} \%*\% \text{Const1} \]

\[ s21 <- Z[1,2] \]

\[ s12 <- s21 \]

\[ s22 <- Z[2,2] \leftarrow \text{Asset mean} \%*\% \text{Inverse Covariance} \%*\% \text{Asset mean} \]

\[ \text{NewVec} = \text{c}(2, 2*\text{Investor rrr}) \]

#Finding the Lagrangians

\[ u <- \text{solve(Z, NewVec)} \]

#Now, the weights are the only left unknown variables. Calculating, we get the following:

\[ \text{Weight Vec} \leftarrow \text{Inverse Covariance} \%*\%(u[1]*\text{Const1} + u[2]*\text{Asset mean})/2 \]

#Printing the Optimal Weights obtained from the Lagrange Multipliers Method Calculated above

\[ \text{Weight Vec} \]

#CORRESPONDING PORTFOLIO RISK USING CROSS PRODUCT OF THE WEIGHTS

\[ \text{risk portfolio} \leftarrow \text{t(Weight Vec)} \%*\% \text{Covariance Matrix} \%*\% \text{Weight Vec} \]

\[ \text{Std portfolio} \leftarrow \text{sqrt(risk portfolio)} \]

#BUILDING THE EFFICIENT FRONTIER

\[ \text{Exp Return Portfolio} = \text{c(rep.int(0,\ P_NUMB))} \]

\[ \text{Std Dev Portfolio} = \text{c(rep.int(0,\ P_NUMB))} \]

\[ x <- \text{matrix(c(rep.int(0, N*P_NUMB)), nrow=N)} \]

\[ y <- \text{matrix(c(rep.int(0,2*P_NUMB)), nrow=2)} \]

\[ z <- \text{matrix(c(rep.int(0,2*P_NUMB)), nrow=2)} \]

#Creating a for loop to avoid redundance

\[ \text{for(j in 1:P_NUMB)} \]

\[ \{ \]

\[ \text{Exp Return Portfolio}[j] <- j*Maximum return/P_NUMB \]

\[ z[,j] <- \text{c}(2, 2*\text{Exp Return Portfolio}[j]) \]

\[ y[,j] <- \text{solve(Z, z[,j])} \]

\[ x[,j] <- \text{Inverse Covariance} \%*\%(y[1,j]*\text{Const1} + y[2,j]*\text{Asset mean})/2 \]

#Standard dev for every j

\[ \text{Std Dev Portfolio}[j] <- \text{sqrt(t(x[,j]) \%*\% Covariance Matrix \%*\% x[,j])} \]

\[ \} \]

#Finally, plotting the efficient Frontier

\[ \text{plot(Std Dev Portfolio, Exp Return Portfolio, main ="Efficient Set.", col = "black")} \]