BINOMIAL OPTION PRICING MODEL TO PREDICT CAC 40 INDEX

FINAL REPORT

April 30th, 2019

Wiam Benhammou

Supervised by Dr. L. Laayouni

BINOMIAL OPTION PRICING MODEL TO PREDICT CAC 40 INDEX
Capstone Report
I, Wiam Benhammou, hereby affirm the following: “I have applied ethics to the design process and in the selection of the final proposed design. And that, I have held the safety of the public to be paramount and have addressed this in the presented design wherever may be applicable.”

_____________________________________________________
Wiam Benhammou

Approved by the Supervisor

_____________________________________________________
Dr. L. Laayouni
ACKNOWLEDGEMENTS

I would like to thank the people who helped throughout this learning experience to better my work and build on it gradually. First of all, I would like to thank my mother along with my brother and my sister for giving me the energy to continue; especially after I lost my dear father at the very beginning of this project. Moreover, I would like to thank Dr. Laayouni Lahcen for his extremely valuable pieces of advice while choosing the topic along with his directions throughout the whole project implementation. Also, I would like to thank my advisor Dr. Nisar Naeem Sheikh for his endless support and inspiration. Last but not least, a special thanks to my friends who reassured and motivated me whenever needed.

Dedicated to the memory of my father, Benhammou Mohamed, who always believed in my potential to be successful. You are gone, but your belief in me and continuous encouragements have made all of this experience possible.
# Table of Contents

ABSTRACT ........................................................................................................................................ 8  
RESUME .......................................................................................................................................... 9  
1. INTRODUCTION .......................................................................................................................... 10  
   1.1. Background: ......................................................................................................................... 10  
   1.2. Capstone Project Objective ................................................................................................. 10  
   1.3. Report Outline ..................................................................................................................... 10  
2. METHODOLOGY .......................................................................................................................... 11  
   2.1. Introducing Concepts: ........................................................................................................... 11  
      2.1.1. History of Options: ........................................................................................................ 11  
      2.1.2. Options: ........................................................................................................................ 12  
      2.1.3. Stock Options: .............................................................................................................. 12  
      2.1.4. Index Options: ............................................................................................................. 12  
      2.1.5. Call Options: ............................................................................................................... 13  
      2.1.6. Put Options: ................................................................................................................. 13  
      2.1.7. American VS European VS Capped Options: .............................................................. 13  
3. CAC 40 Index Data Background and Analysis ............................................................................. 14  
   3.1. History of CAC 40 Index: ....................................................................................................... 14  
   3.2. CAC 40 Index Descriptive Statistic: ..................................................................................... 15  
      3.2.1. CAC 40 Index Central Tendency: ............................................................................... 15  
      3.2.2. CAC 40 Index Dispersion: ......................................................................................... 16  
   3.3. CAC 40 Index Chart Analysis: ............................................................................................ 17  
4. Binomial Option Pricing Model ..................................................................................................... 19  
   4.1. Options Pricing: .................................................................................................................... 19  
   4.2. Binomial Option Pricing Model: .......................................................................................... 19  
   4.3. Model Rules and Assumption: ............................................................................................ 20  
   4.4. Effect of Risk-Free Interest Rate on the Binomial Option Pricing Model: ....................... 21  
   4.5. Effect of Stock Volatility on the Binomial Option Pricing Model: .................................... 22  
5. Model Implementation on CAC 40 Index Using R ......................................................................... 22  
   5.1. R Programming Language ................................................................................................... 22  
   5.2. Binomial Option Pricing Model Build-up on R .................................................................... 22  
      5.2.1. Stock Tree .................................................................................................................... 22  
      5.2.2. Option Price Tree ....................................................................................................... 26  
      5.2.3. One Period on CAC 40 Index .................................................................................... 28
5.2.4. Two Period on CAC 40 Index ................................................................. 28
5.2.5. N Periods on CAC 40 Index ................................................................. 29
5.2.6. Time Complexity and Parallel Computing ........................................... 30

6. STEEPLE Analysis...................................................................................... 32
   6.1. Social & cultural factor ......................................................................... 32
   6.2. Technological factor .............................................................................. 32
   6.3. Economic factor ................................................................................... 32
   6.4. Environmental factor ........................................................................... 33
   6.5. Political factor ..................................................................................... 33
   6.6. Legal & legislative factor ....................................................................... 33
   6.7. Ethical factor ....................................................................................... 33
   6.8. Future Work ....................................................................................... 33

7. CONCLUSION .............................................................................................. 34

8. REFERENCES ............................................................................................ 35

9. APPENDIX A ........................................................................................... 37
List of Figures & Tables

Table 1. CAC 40 Index Central Tendency Measures
Figure 2. CAC 40 Index Variables Box Plot
Figure 3. CAC 40 Index Volume Box Plot
Table 4. CAC 40 Index Box Plot Results
Table 5. CAC 40 Index Variance Results
Figure 6. CAC 40 Index Bar Chart
Figure 7. Bar Chart with Add INS
Figure 8. CAC 40 Index Last Quarter
Figure 9. Binomial Stock Price Tree
Figure 10. Binomial Stock Tree
Figure 11. FCHI Adjusted Values Density Plot
Figure 12. FCHI Adjusted Values Scatter Plot
Figure 13. FCHI Volatility Using the Closing Price Method
Figure 14. Method 1 Volatility Density Plot
Figure 15. Method 1 Volatility Scatter Plot
Figure 16. FCHI Volatility Using the Garman Klass Method
Figure 17. Method 2 Volatility Density Plot
Figure 18. Method 2 Volatility Scatter Plot
Figure 19. Risk Neutral Probability
Figure 20. Binomial Option Tree
Figure 21. Compute the Number of Stocks in CAC 40 Portfolio
Figure 22. Inclusive Binomial Option Pricing Model
Figure 23. Binomial Option Pricing Model- One Period on CAC 40 Index
Figure 24. Call Option Pricing Result
Figure 25. Put Option Pricing Result
Figure 26. Binomial Option Pricing Model- Two Periods on CAC 40 Index
Figure 27. Call Option Pricing Result
Figure 28. Put Option Pricing Result
Figure 29. Binomial Option Pricing Model- First Sequence of Periods
Figure 30. Binomial Option Pricing Model - Second Sequence of Periods
Figure 31. Sequence 1 Call Value Plot
Figure 32. Sequence 2 Call Value Plot
Figure 33. Running time on R on Different Number of Periods
Figure 34. Parallel Computing on R
Figure 35. Parallel Computing Running Time on the Model
Figure 36. CAC 40 Index Data Frame on R
Figure 37. CAC 40 Index Symbol on R Environment
Figure 38. CAC 40 Index Descriptive Statistic on R
Figure 39. CAC 40 Stock Tree
Figure 40. CAC 40 Binomial Option Pricing Model
ABSTRACT

The following work is an in depth report of my capstone project about the Binomial Option Pricing Model to Predict CAC 40 Index. The model deploys the mathematical binomial coefficients in order to mimic CAC 40 stock movements, predict its value, and value its options eventually. First, the work’s methodology starts with an introduction of the model’s background along with its development and main applications throughout history. To ensure a clear understanding of the work, main definitions of the model’s technical terms are provided. Second, a detailed statistical analysis of CAC 40 index data and background is given to relate the model’s technical results with the social, political, and economical events the French market has gone through during 2018. Afterwards, the binomial option pricing model was implemented on R language in order to price CAC 40 options on three cases: one period, two periods, and a sequence of periods. The results proved to be close to the expected values and in case of sequential periods the binomial option pricing model tends to behave like the Black & Scholes model. Moreover, an evaluation of the model will be done to assess its strengths and weakness based on memory and time complexity. Last but not least, the project’s STEEPLE analysis will be presented to evaluate it with respect to different factors, besides a discussion about the future work intended to develop later.

Keywords: Binomial Option Pricing Model, CAC 40 Index, Binomial Coefficient, Sock, R Language;
RESUME


1. INTRODUCTION

1.1. Background:
The vast majority of investments involve a great deal of risk. If there was a way to know all of the parameters determining stocks fluctuations in an accurate way, stock’s predictions would be a determined process and market uncertainty would vanish. Unfortunately, this is not possible. However, financial analyst, mathematicians or even physicians have worked on models throughout history to better financial predictions with the objective of increasing the profit and decreasing the risks. One of the models heavily used in the market are the ones concerned with: valuation of options. To mention that such models imply estimations in some of their parameters which affect their accuracy. As a result, a lot of work is done on such models to help in their development in order to improve their performance gradually. One of these models is the Binomial Option Pricing Model, which is frequently used for option pricing besides the Black & Scholes Model.

1.2. Capstone Project Objective
Throughout this capstone project, a series of steps will be presented in order to build the binomial option pricing model. The goal is to have a detailed overview of the model, its main parameters, strengths, and weaknesses. Besides, an application of the model will be introduced and discussed methodically. The data set on which the model will be applied and tested is CAC 40 index during 2018, which is a strong barometer of the French market health. The model will be assessed during different periods. First, from a result point of view and then from both time and memory complexity perspective. Afterwards, the model will also be compared to the Black and Scholes model for accuracy.

1.3. Report Outline
This project follows a methodical outline to give a detailed steps of the binomial option pricing model to predict CAC 40 index. The first chapter provides an overview of the project. Afterwards, the second chapter gives an introduction and explanation of the model’s main technical terms like options, stock options, index options, and types of options. The third chapter
deals with a comprehensive historical background and statistical analysis of CAC 40 index data, over the year of 2018, using the statistical language R. The fourth chapter gives a set of rules and assumptions respected to build the model, besides the effect of the interest rate and volatility on the model performance. The fifth chapter tackles the model build-up using the R programming language followed by an analysis of the results generated in the three cases on the index. Furthermore, a STEEPLE analysis is introduced to assess the project with respect to different types of factors, along with a discussion about the potential future work and how to expand and better the performance of the model in the future. Last but not least, a conclusion will sum up the work besides its results by underlining the main highlights and findings of the project.

## 2. METHODOLOGY

### 2.1. Introducing Concepts:

#### 2.1.1. History of Options:

Options can be traced back in history to the ancient Greek era. In his book “Politics”, the great philosopher Aristotle wrote about the way the mathematician, astronomer, and philosopher Thales of Miletus managed to make a remarkable profit from “olive harvest” at that time [1]. Based on his observations and analysis, Thales was able to predict whether there would be a huge olive harvest in a specific year, based on which he made profit. In fact, in case of a potential large olive harvest, he would know that the demand of olive presses would increase. Consequently, he used to pay a certain amount of money to olive presses proprietors, ahead of time, to guarantee the rights to exploit them at the expected large harvest times. Following his predictions, Thales would sell his rights on the presses at higher prices and made a great deal of profit eventually. And this has, in fact, marked the first practice of a call option in the recorded history [1]. Another additional example of the historicity of options is illustrated in the Tulip Mania Bubble. During the 17th century, there was an extremely insane demand of Tulip Mania Bubble. During the 17th century, there was an extremely insane demand of Tulip flowers in Netherland for their distinguished social status and prestigious symbolic significance. As a result, Tulip Bubble was born [2]. The flower’s price skyrocketed and options were massively used by the Tulip’s cultivators and wholesalers relatively to protect themselves from the market uncertainty; i.e. the business risks [2]. At first options were frequently traded over-the-counter; just between the two parties, to avoid
public transactions on the exchange. Further in time, options trading started to be regulated and controlled. This latter was done, officially in 1973, by the biggest exchange for commodity and contracts in the world “Chicago Board of Trade” [3]. From then on, options have been traded massively. On April 2019, and according to the “Options Clearing Corporation”, the total exchange listed options surpassed 4 billion contracts with 39 million contracts on average of index options [4].

2.1.2. Options:

Option is a special listed financial security, which involves pure bets between two parties, with some special characteristics that differentiate it from the other known securities. In other words, it is a contact between a buyer and a seller that gives to the buyer the right, but not the obligation, to exercise it up to a previously specified date. Exercising an option is done based on the agreed upon price between the two parties. Simply put, options give to their holders more flexible options in their investment situations. The payoffs of options are never negative, this is why they are very profitable, if used accurately and pragmatically. For example the proprietors can protect their assets and insure their profit in different market cases (whether it is a rise or a fall of prices) [3]. On a daily basis, options are frequently used. For example, at my Moroccan college Al Akhawayn University in Ifrane, students have the right to drop a course up to a definite and previously known deadline. To mention that students have the right to drop a course but not the obligation to do so, and the price of this option was initially included in their tuition fees.

2.1.3. Stock Options:

Stock options are, usually called equity options, elemental securities in which the asset in question is shares of stocks. They give the right to the parties involved to buy or sell 100 shares per contract [5]. The buyer of the option pays a “premium” to the seller and interchangeably the seller guarantees the buyer the right to sell or buy the stock at a fixed price, also sometimes called a strike price, up to an expiration date. After that date, stock options’ value disappears automatically. To mention that there exist two types of options: call options and put options [5].

2.1.4. Index Options:

Principally, an index is a statistical indicator of the change happening in a specific market. A stock index expresses the overall behavior of a set of stocks. Following the same logic, an index option represents its option on the index of the stocks in question. Some of the most reputable world market indices are the American S&P 100 along with STANDARD &
POOR’s 500 (generally known as S&P’s 500), Japanese NIKKEI 225, French CAC 40, and British FTSE 100 [6]. Investors prefer to trade on index options, which is more time efficient, rather than the huge number of stocks separately. Moreover, and thanks to index options, trading can be done without proceeding with stock delivery using what is called ‘‘cash settlement’’ [6]. The value of cash settlement is arbitrated based on the value of the index at the end of the day instead of the index value when the exercise was arranged [3].

2.1.5. Call Options:

The call option deals with the case of having the option to buy the stock at a fixed price. Call option holders are likely to have some optimistic predictions about the stock prices. With that been said, in case of a rise in stock prices, the value of the call option increases. Consequently, old call options will be sold for greater prices, and the same goes for the new call options which will be sold for greater premiums [3].

The call option is said to be [3]:

I. In the money; if the stock price exceeds the exercise price
II. Out of the money; if the stock price is less than exercise price
III. At the money; if the stock price is the same as the exercise price

2.1.6. Put Options:

The put option is about having the option to sell the stock at a fixed price. Inversely, put option holders have some pessimistic expectations of the stock prices. Therefore, the value of the put option increases if and only if the stock prices go down. As a result, old put options will be sold for greater prices, and the new put options will be sold for greater premiums. To mention that high orders to buy put options is a barometer of a potential downturn in the market [3].

The put option is said to be [3]:

I. In the money; if the stock price is less than the exercise price
II. Out of the money; if the stock price exceeds the exercise price
III. At the money; if the stock price is the same as the exercise price

2.1.7. American VS European VS Capped Options:

The three relatively known styles of options are different based on their exercise date. On one hand, the proprietor of the European option (does not mean that it is only used in Europe) enables him or her to exercise it only at the maturity/expiration date. On the other hand, the American option is more flexible; since it gives the right to its holder to exercise it anytime
and up to its expiration date, hence it is relatively highly priced comparing to the price of a European option. The last and the least known option style is the capped one. Capped option grants to the proprietor the possibility to exercise it only during a specific period of time before its maturity date [5].

3. CAC 40 Index Data Background and Analysis

3.1. History of CAC 40 Index:

The index CAC 40 stands for “Cotation Assistée en Continu” which means continuous assisted trading in English. It is a financial measurement used efficiently to understand the French market and to assess and weigh the returns on diverse investments [7]. The index is economically significant, in Paris stock market, similarly to the three major American stock indices which are ‘NASDAQ Composite’, ‘Dow Jones Industrial Average’ along with ‘STANDARD & POOR 500’ [7].

According to the ‘Pan-European Euronext’ official website, CAC 40 index was initially created on the last day of December 1987 with a base value of 1000 points [10]. The index is a weighted one; since it is important to avoid disproportionate influence on the index in the situation of huge companies with few number of the shares made public [10]. Moreover, the index is based on the ‘free-float adjusted market capitalization’. According to the principle of ‘free-float adjusted market capitalization’, the top 100 companies are reviewed quarterly and 40 of them are chosen eventually to join the CAC 40 index [10].

Below is the latest list of the CAC 40 index components companies during March of 2019 published on the Euronext official website:

“ACCOR, AIR LIQUIDE, AIRBUS, ARCELORMITTAL SA, ATOS, AXA, BNP PARIBA ACT.A, BOUYGUES, CAPGEMINI, CARREFOUR, CRÉDIT AGRICOLE, DANONE DASSAULT SYSTÈMES, ENGIE, ESSILOR LUXOTTICA, HERMES INTL, KERING L’ORÉAL, LEGRAND, LVMH, MICHELIN, ORANGE, PERNOD RICARD, PEUGEOT PUBLICIS GROUPE SA, RENAULT, SAFRAN, SAINT GOBAIN, SANOFI, SCHNEIDER ELECTRIC, SOCIÉTÉ GÉNÉRALE, SODEXO, STMICROELECTRONICS, TECHNIP
FMC TOTAL, UNIBAIL-RODAMCO-WE, VALEO, VEOLIA ENVIRON, VINCI, VIVENDI.” [9].

3.2. CAC 40 Index Descriptive Statistic:

The descriptive statistics will be based on one year of CAC 40 index data, from the 1st of January 2018 until the 31st of December 2018. The CAC 40 index data was extracted from Yahoo Finance website and is considered statistically and strongly reliable for any type of analysis. Every stock market index in the world has its own and unique symbol. With that been said, CAC 40 stock index symbol is FCHI. FCHI and CAC 40 will used interchangeably in the model throughout this project.

3.2.1. CAC 40 Index Central Tendency:

CAC 40 index dataset contains six variables, which represent the index: Open, Close, High, Low, Adjusted, and Volume points values.

The following table summarizes CAC 40 index parameters’ values with respect to the minimum, median, mean and maximum functions as central of tendency measures:

<table>
<thead>
<tr>
<th>FCHI Index Points</th>
<th>FCHI.Open</th>
<th>FCHI.High</th>
<th>FCHI.Low</th>
<th>FCHI.Close</th>
<th>FCHI.Volume</th>
<th>FCHI.Adjusted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>4641</td>
<td>4664</td>
<td>4556</td>
<td>4599</td>
<td>23249900</td>
<td>4599</td>
</tr>
<tr>
<td>Median</td>
<td>5345</td>
<td>5367</td>
<td>5323</td>
<td>5344</td>
<td>82529900</td>
<td>5344</td>
</tr>
<tr>
<td>Mean</td>
<td>5297</td>
<td>5322</td>
<td>5268</td>
<td>5294</td>
<td>85230645</td>
<td>5294</td>
</tr>
<tr>
<td>Max</td>
<td>5638</td>
<td>5657</td>
<td>5629</td>
<td>5640</td>
<td>222510800</td>
<td>5640</td>
</tr>
</tbody>
</table>

Analysis: From 01/01/2018 until 01/01/2019 CAC 40 Index Closing value was on average about 5294 points. However, the year of 2018 did not end on good terms; the index has witnessed its worst performance since 2011 with a value of 4730.69 points, on the 31st of December 2018, which represents a fall of exactly 10.95% [11]. In fact, 2018 was a relatively dark year for the Parisian market due to a set of reasons. To mention mainly the political factors involving the uncertainty around the Brexit and the American government shut down, along with the trade tensions between the United States and China; also called as “US-China trade war” [11].
3.2.2. CAC 40 Index Dispersion:

![Figure 2. CAC 40 Index Variables Box Plot](image1)

![Figure 3. CAC 40 Index Volume Box Plot](image2)

**Analysis:** Above are figures representing the box plot of the CAC 40 index: Open, High, Low, Close, Adjusted, and Volume values throughout the year of 2018. The lower outliers in the figure to the left (Figure 2) represents more than 3/2 times of the lower quartile. However, CAC 40 index volume (Figure 3) knew in the year of 2018 numerous upper outliers which represent more than 3/2 times of the upper quartile; and that is a sign of a volatile market. The following table summarizes the box plot results of all the six variables:

<table>
<thead>
<tr>
<th>Quartile</th>
<th>FCHI.Open</th>
<th>FCHI.High</th>
<th>FCHI.Low</th>
<th>FCHI.Close</th>
<th>FCHI.Adjusted</th>
<th>FCHI.Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>25%</td>
<td>5148.710</td>
<td>5176.66</td>
<td>5115.625</td>
<td>5148.315</td>
<td>5148.315</td>
<td>71796150</td>
</tr>
<tr>
<td>50%</td>
<td>5344.550</td>
<td>5366.56</td>
<td>5322.770</td>
<td>5344.260</td>
<td>5344.260</td>
<td>82529900</td>
</tr>
<tr>
<td>75%</td>
<td>5472.835</td>
<td>5491.81</td>
<td>5448.595</td>
<td>5473.345</td>
<td>5473.345</td>
<td>94334300</td>
</tr>
</tbody>
</table>

The dispersion of the index can be also calculated based on the variance of its components. The table below summarizes CAC 40 Index variance over the studied period. To mention that the variance will be further analyzed in this project as a measure of index volatility.
### Table 5. CAC 40 Index Variance Results

<table>
<thead>
<tr>
<th>Index</th>
<th>FCHI.Open</th>
<th>FCHI.High</th>
<th>FCHI.Low</th>
<th>FCHI.Close</th>
<th>FCHI.Adjusted</th>
<th>FCHI.Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance</td>
<td>214.6629</td>
<td>211.8273</td>
<td>214.0135</td>
<td>219.0871</td>
<td>219.0871</td>
<td>24039911</td>
</tr>
</tbody>
</table>

#### 3.3. CAC 40 Index Chart Analysis:

*Figure 6. CAC 40 Index Bar Chart*

*Figure 7. Bar Chart with Add INS*

**Analysis:** The figures above show CAC 40 index candlestick charts. Each candlestick represents a single day in the life of the CAC 40 stocks. The green color indicates that the price of the stock ended that day higher than the previous day, whilst the orange color indicates that the price of the stock ended that day lower than the previous day. Very long candlesticks of the same day are signs of volatile stocks (independently of the color). As was previously mentioned, during December 2018 the index has experienced its worst record since
2011. The figure 6 shows long orange candlesticks starting October 2018; which indicates overall downs. The blue graph at the bottom represents RSI, i.e. the Relative Strength Index. The RSI in the case of CAC 40 index has experienced in 2018 more oversold periods (less than 40%) than overbought ones (more than 70%). In 2018 CAC 40 relative strength index was equal to 41.141; which does not reflect an overall healthy market.

Meanwhile, the figure 7 illustrates a bar chart of CAC 40 index with MACD and BBands add INS. On one hand, the MACD stands for “Moving Average Convergence and Divergence”; and it is a powerful indicator of a bearish or a bullish market. In the case of CAC 40 index, the behavior of the two moving averages, represented by the gray and the dotted red lines, are indicator of how stock prices change over time. The MACD can anticipate the behavior of stock prices, as seen starting October 2018, the dashed dotted red line is above the gray line indicates a clear drop in the stock prices which was the case [16]. On the other hand, the BBands, which stands for “Bollinger Band”, the instrument is illustrated as two standard deviation lines plotted around the stock's movement over time. BBands are good indicator of the volatility of the market, and in the case if CAC 40 index, it is clear that the distance between the lines is wide enough to presume high volatility. In fact, the CAC 40 index high volatility in 2018 was a result of mainly the economic and political factors the European countries in general, and France more precisely, were facing; which increased the overall market uncertainty [17].

CAC 40 index will be analyzed further, it will be used to apply Binomial Option Pricing Model in the upcoming work.
Analysis: CAC 40 index is examined every four months by a special committee that is the “Conseil Scientifique” in French. Figure 8 represents a bar chart of the last quarter in the year of 2018, which demonstrates clearly the downs the index experienced. In the 3rd of January 2019 the index behavior went down to a value of 4611.49 followed by a rise and then a drop to 4719.17 which is lower than the previous drop. Consequently, this illustrates an “Inverse Head and Shoulders” pattern. This latter indicates that there will be a future rise in the stocks, which was the case of CAC 40 after the 7th of January of 2019 [18].

Figure 8. CAC 40 Index Last Quarter

4. Binomial Option Pricing Model

4.1. Options Pricing:

Buying options can simulate the act of buying insurance. Options are a way to protect investors’ position against the market fluctuations caused by uncertainty. Since insurance is expensive in the financial market, and following the same logic options are expensive too. This is why it was very important to find ways to price options. Consequently, and in order to determine the theoretical fair value of an option, there exist different models, to mention: “Black & Scholes Model”, “Binomial Option Pricing Model”, and “Trinomial Model” [3].

4.2. Binomial Option Pricing Model:

The binomial option pricing model was primarily introduced on 1979 by Cox, Rubinstein, and Ross [19]. The model is not based on a mathematical formula like the one of Black & Scholes. In fact, BOPM is built on the principle of Bernoulli trial with two possible outcomes.
In the case of stock options, the stock price can either go up or down; which is practically governed by the binomial’ discrete, probability distribution [19]. The binomial option pricing model has proven, up to now, to be one of the simplest option pricing models, yet it incorporates extraordinary deep economical results. The model adopts the binomial distribution primary conditions; which include the following: First, the possible outcomes are independent from each other, and second, the stock can either go up to a specific value or down by a specific value but never both at the same time [20].

The Binomial Option Pricing Model simulates the life of the option as a series of discrete points in time depending on a previously determined periods. Below the figure illustrates the movements of the stock price following the binomial tree principal.

Analysis: based on today’s stock price denoted as $S$, which is the head of the tree, the movement of the stock price is restricted to only two possibilities. $S$ can go up to a value of $U$ or can go down by a value of $D$ from today until the determined expiration date. To mention that $U$ and $D$ are the inverse of each other. Thus the recombination of tree’s leaves after two periods in each level.

![Figure 9. Binomial Stock Price Tree](image)

4.3. Model Rules and Assumption:

To proceed with Binomial Option Pricing model, the following set of rules and assumptions need to be put:

a. Today stock price is: $S_0$

b. $T$ is the expiration date, with $N$ number of periods

c. $\Delta t$ is the time represented in each period

d. If the stock price goes up with a value $U$; such that: $U = e^{\sigma \sqrt{\Delta t}}$

the new stock price value is: $S_u = S_0 * U$

e. If the stock price goes down by a value $D$; such that: $D = e^{-\sigma \sqrt{\Delta t}}$

the new stock price value is: $S_d = S_0 * D$
f. The relationship between U and D is as follow: $U = 1/D$

g. The risk free interest rate, $r$, is constant.

h. The standard deviation, $\sigma$ sigma, of the returns of the stock is the same annually.

Because of the fact that the movement of the stock price is restricted to two cases, it goes either up or down with a certain value. And given the state price in both cases $p$ for up and $q$ for down for an $n$ exercise time period; a European put and call option prices are calculated based on the binomial coefficient as follow [15]:

$$
C(X) = \sum_{j=0}^{n} p^j \cdot q^{n-j} \binom{n}{j} \quad \text{Max}[S \cdot U^j D^{n-j} - X, 0]
$$

$$
P(X) = \sum_{j=0}^{n} p^j \cdot q^{n-j} \binom{n}{j} \quad \text{Max}[X - S \cdot U^j D^{n-j}, 0]
$$

Such that the binomial element is calculated as follow:

$$
\binom{n}{j} = \frac{n!}{j! (n-j)!}
$$

4.4. Effect of Risk-Free Interest Rate on the Binomial Option Pricing Model:

One of the important variables needed to incorporate into the computational part of the model is: the risk-free interest rate. It is known that the interest rate is not a fixed constant in real life financial stock markets. Consequently, an approximation should calculated or an assumption should be made. In fact, in case of over estimations, call premiums are projected to be higher and put premiums to be lower than their real values. The Binomial Option Pricing Model assumes a constant value of the asset. In case of stocks; the value depends on the index [21]. Therefore, two main assumptions are made. First, the actual value of the stock is the same as its predicted payoff deducted at the risk-free rate. Second, arbitrage opportunities are assumed to not exist in this case; eliminating the possibility of finding the exact same stock priced differently in two or more markets [22].
4.5. Effect of Stock Volatility on the Binomial Option Pricing Model:
Volatility is the extent to which the stock returns are dispersed over a determined time frame. In fact, volatility is a major input of the Binomial Option Pricing Model and is considered as the trickiest variable comparing to the other model parameters. Stock index volatility can be measured based on the historical data of its standard deviation by using the closing prices of the index over the desired time frame [14]. In case of wrong volatility assumptions options pricing accuracy is expected to drop.

5. Model Implementation on CAC 40 Index Using R

5.1. R Programming Language
R programming language is a strong software frequently used in statistics, data processing, and data visualization in general. More precisely, the tool is highly applicable in the finance field and used to build models, forecast, and implement in depth financial analysis. Even if it is considered relatively as a recent language (1993) comparing to the other ones already used in such areas, it has been evolving rapidly during the past years. R language is favored over other data-analysis programming tools thanks to its flexibility, strong community, along with its high computing performance [12].

5.2. Binomial Option Pricing Model Build-up on R
The model will be built on CAC 40 data over the year of 2018, which is from the 1st of January until the 31 of December. Since CAC 40 options are exercised only at the expiration date- also sometimes called the maturity date, they are considered to be European options. Consequently, the binomial option pricing model will be built on R accordingly. The index varied behavior over time is eventually just a reflection of stocks fluctuations. To recall that the model assumes that the stock price can either go up or down with specific values following a Geometric Brownian Motion.

5.2.1. Stock Tree
In order to build the binomial option pricing model, the first step is actually to build a stock tree; which mimics the two possible outcomes of a stock movement over time.
Below is the function used to build the stock tree on R:

```r
StockTree = function(S, Sigma, DeltaT, N) {
  tree = matrix(0, nrow= N+1, ncol= N+1)
  #calculate the value by which the stock can go up OR down (Geometric Brownian Motion)
  u = exp(Sigma*sqrt(DeltaT))
  d = exp(-Sigma*sqrt(DeltaT))
  #loop on the number of period simulated
  for (i in 1:(N+1)) {
    for (j in 1:i) {
      tree[i,j] = S * u^(j-1) * d^((i-1)-(j-1))
    }
  }
  return(tree)
}
```

*Figure 10. Binomial Stock Tree*

As it is illustrated on the *figure 10* the binomial stock tree requires as inputs the current stock price S, the stocks standard deviation Sigma, the amount of time DeltaT in each previously determined number of periods N. On one hand, the stock price is considered as the adjusted closing price instead of the closing price. In fact, the adjusted price tends to give a more accurate idea of the behavior of the stocks; since it takes into consideration the overall corporates’ actions [23].

Below are the figures illustrating CAC 40 index adjusted values over a period of one year:

*density.default(x = FCHIAdjusted)*

*Figure 11. FCHI Adjusted Values Density Plot  Figure 12. FCHI Adjusted Values Scatter Plot*
On the other hand, Sigma is the most relatively hard to determine accurately among the other inputs of the function. To compute Sigma—denoted sometimes as volatility, two methods will be examined in order to choose the best one to apply on CAC 40 index data. The two chosen method are: volatility based on the closing price and volatility based on Garman Klass.

To start with CAC 40 index volatility using the Closing price method.

Below the figure 13 demonstrates the lines of code executed on R to compute the index volatility:

```r
# volatility calculation based on the CLOSING price method
volatility <- as.data.frame(volatility(FCHI, n = 10, N = 255, calc = "close", mean0 = FALSE))
VolatilityNumeric <- Volatilitydf[,]
CAC40Volatility1 <- na.omit(VolatilityNumeric) # omitting missing values
t1 <- c(1:length(CAC40Volatility1))
plot(CAC40Volatility1 ~ t1) # Volatility density plot
plot(density(CAC40Volatility1))
```

**Figure 13. FCHI Volatility Using the Closing Price Method**

Data visualization is a powerful tool to easily compare the two methods. Consequently, below are the two figures illustrating initially the closing price volatility distribution in two plots:

**Figure 14. Method 1 Volatility Density Plot**

**Figure 15. Method 1 Volatility Scatter Plot**

Second, CAC 40 index volatility using the Garman and Klass method. Below is the figure that illustrates the lines of code executed on R in order to compute the index volatility.
Figure 16. FCHI Volatility Using the Garman Klass Method

Below are the figures illustrating the Garman Klass price volatility distribution in two plots:

Analysis: The two methods generate different results. As shown in the above density plots, and for the same number of index entries $N = 246$, the density plot bandwidths are different. In fact, the more the distribution is smoothened the bigger the bandwidth becomes. Since, Garman and Klass bandwidth is smaller than the one of Closing price method, the Garman Klass is considered to be more representative of the index volatility. In fact, Garman and Klass estimator proved to be about seven times more efficient than Closing price estimator; and this is a result of the method’s assumption of Brownian motion with a drift equals to zero excluding jumps in the opening [13]. As a result, and since the objective is to have an efficient binomial option pricing model, the Garman and Klass method will be used for volatility estimation throughout the upcoming model build up steps.
5.2.2 Option Price Tree

After building the stock tree, options price can be generated systematically from it. In case of a Call; the option price would be the maximum value between 0 and (stock - exercise) price. Inversely, in case of a Put; the option price would be the maximum value between 0 and (exercise - stock) price. Thus:

\[
\text{Call Option Price} = \text{Max}((Su/d - X), 0)
\]

\[
\text{Put Option Price} = \text{Max}((X - Su/d), 0)
\]

To build up the call/put option price tree the risk neutral probability should be taken into consideration [14]. Following are the steps needed to identify the option’s price:

1. Construct a portfolio which mimics the variation of the stock, which will consequently have the same payoff if the stock price increases/decreases.
2. Deduct the portfolio’s payoff based on the risk-free rate.
3. Derive systematically the option price

To mention that one of the model’s assumptions is that arbitrage is not allowed in this case and that investors deal with the risk-neutral rate as if options are riskless investments [14]. As a result, the risk neutral probability is calculated as follow:

\[
\alpha = (R - d) / (u - d)
\]

Such that:

\[
\alpha : \text{is the risk neutral probability}
\]

\[
d : \text{is the value by which the stock price can go down}
\]

\[
u : \text{is the value by which the stock price can go up}
\]

\[
R : \text{is the discount element}; \quad R = \exp(r \times \Delta t)
\]

The option price is then calculated as follow:

\[
\text{Put option:} P = (\alpha \times P_u + (1 - \alpha) \times P_d) / (R)
\]

\[
\text{Call option} \quad C = (\alpha \times C_u + (1 - \alpha) \times C_d) / (R)
\]

Below are the functions created on R to compute the risk neutral probability \(\alpha\) along with the binomial option tree handling the case of a put option and a call option inclusively:

```r
alphaProbability = function(r, DeltaT, Sigma) {
  u = exp(Sigma*sqrt(DeltaT))
  d = exp(-Sigma*sqrt(DeltaT))
  return((exp(r*DeltaT) - d) / (u-d))
}
```

*Figure 19. Risk Neutral Probability*
Now that the binomial option tree is all set, it can be used on CAC 40 index data. However, and in order to completely mirror CAC 40 index portfolio another variable should be determined and that is: the number of stocks in that portfolio $\Delta$ [15]. Such that:

$$
\Delta = \frac{C_u - C_d}{S_u - S_d}
$$

Consequently the R code below build up the binomial option pricing model with the following input as a list: stock tree, option tree, risk neutral probability value, along with the number of shares needed to mirror the portfolio.

```r
BOTree = function(tree, Sigma, DeltaT, r, X, type) {
  alpha = alphaProbability(r, DeltaT, Sigma)
  OptionTree = matrix(0, nrow=nrow(tree), ncol=ncol(tree))
  if(type == 'call') {
    OptionTree[nrow(OptionTree),] = pmax(tree[nrow(tree),] - X, 0)
  } else if(type == 'put') {
    OptionTree[nrow(OptionTree),] = pmax(X - tree[nrow(tree),], 0)
  }
  for (i in (nrow(tree)-1):1) {
    for(j in 1:i) {
      OptionTree[i,j] = (1-alpha)*OptionTree[i+1,j] + alpha*OptionTree[i+1,j+1]/exp(r*DeltaT)
    }
  }
  return(OptionTree)
}
```

**Figure 20. Binomial Option Tree**

```
delta = function(BOM, row, col) {
  StockTree = BOM$CAC40_Stocks
  OptionTree = BOM$CAC40_Option
  return(
    (OptionTree[row+1, col+1] - OptionTree[row+1, col])/(StockTree[row+1, col+1] - StockTree[row+1, col]))
}
```

**Figure 21. Compute the Number of Stocks in CAC 40 Portfolio**

Consequently the R code below build up the binomial option pricing model with the following input as a list: stock tree, option tree, risk neutral probability value, along with the number of shares needed to mirror the portfolio.

```r
#BOM which includes a generic function that outputs a list of results----------------------
BOM = function(type, Sigma, T, r, X, S, N){
  alpha = alphaProbability(r, DeltaT = T/N, Sigma = Sigma)
  CAC40tree = StockTree(S = S, Sigma = Sigma, DeltaT = T/N, N = N)
  option = BOTree(CAC40tree, Sigma = Sigma, DeltaT = T/N, r = r, X = X, type = type)
  return(list(CAC40_Stocks = CAC40tree, CAC40_Option = option, Price = option[1,1],
              alpha = alpha, delta = delta))
}
```

**Figure 22. Inclusive Binomial Option Pricing Model**
5.2.3. One Period on CAC 40 Index

The binomial option pricing model can generate options prices on different periods of time. To start with applying the model on CAC 40 index in case of one period $N = 1$ with $r = 2\%$ [10]. Following are the results of call and put European options generated by calling the model built on R:

```r
# ONE PERIOD
BOM(type = 'call', Sigma = CAC40Volatility2[1], T = 1, r = 0.02, X = treeCAC40[1], S = treeCAC40[1], N = 1)
BOM(type = 'put', Sigma = CAC40Volatility2[1], T = 1, r = 0.02, X = treeCAC40[1], S = treeCAC40[1], N = 1)
```

![Figure 23. Binomial Option Pricing Model- One Period on CAC 40 Index](image)

When applying the model on CAC 40 dataset to one period, with:

$S = 5288.60$, $T = 1$, $r = 0.02$, and most importantly number of periods $= 1$, we get stock and option price tree with 2 levels of depth which is illustrated in the following figures for both option types:

![Figure 24. Call Option Pricing Result](image)

![Figure 25. Put Option Pricing Result](image)

5.2.4. Two Period on CAC 40 Index

The second step would be to apply the binomial option pricing model in the case of two periods. With that been said, the dataset is the same but the difference would be calling the function with the input $N = 2$. Following are the results of call and put European options generated by calling the model built on R:

```r
# TWO PERIODS
BOM(type = 'call', Sigma = CAC40Volatility2[1], T = 1, r = 0.02, X = treeCAC40[1], S = treeCAC40[1], N = 2)
BOM(type = 'put', Sigma = CAC40Volatility2[1], T = 1, r = 0.02, X = treeCAC40[1], S = treeCAC40[1], N = 2)
```

![Figure 26. Binomial Option Pricing Model- Two Periods on CAC 40 Index](image)

When applying the model on CAC 40 dataset to two periods, with:
S = 5288.60, T = 1, r = 0.02, and most importantly number of periods = 2, we get stock and option price tree with 3 levels of depth which is illustrated in the following figures for both option types:

\[
\begin{array}{c|c|c|c|}
\text{Period} & \text{S0} & \text{S1} & \text{S2} \\
\hline
1 & 5288.600 & 0.000 & 0.000 \\
2 & 5006.736 & 5586.332 & 0.000 \\
3 & 4739.895 & 5288.600 & 5900.825 \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|}
\text{Period} & \text{C0} & \text{C1} & \text{C2} \\
\hline
1 & 200.495 & 0.000 & 0.000 \\
2 & 0.000 & 350.3542 & 0.000 \\
3 & 0.000 & 0.000 & 612.2248 \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|}
\text{Period} & \text{Price} \\
\hline
1 & 200.495 \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|}
\text{Period} & \text{alpha} \\
\hline
1 & 0.5780153 \\
\end{array}
\]

27. Call Option Pricing Result

5.2.5. N Periods on CAC 40 Index

The last step would be to apply the binomial option pricing model in the case of a sequence of periods. Since all the model’s input parameters are the same, what should be changed is N to a sequence of numbers. Below is the code needed for this case along with the results of the model in two different sequence of periods:

```r
# SEQUENCE OF PERIODS "1"
periods = seq(1, 30)
BOMSeq = function(period) {
  print(period)
  option = BOM(type = "call", Sigma = CAC40Volatility2[1], T = 1, r = 0.02, X = treeCAC40[1], S = treeCAC40[1], N = period)
  return(option$Price)
}

# SEQUENCE OF PERIODS "2"
periods = seq(1, 100)
BOMSeq = function(period) {
  print(period)
  option = BOM(type = "call", Sigma = CAC40Volatility2[1], T = 1, r = 0.02, X = treeCAC40[1], S = treeCAC40[1], N = period)
  return(option$Price)
}
```

Figure 29. Binomial Option Pricing Model- First Sequence of Periods
Figure 30. Binomial Option Pricing Model - Second Sequence of Periods

Plotting the results is the best approach to better visualize the behavior of the binomial option pricing model in the case of large sequential periods N. Following are the two plots of the options’ prices:

![Figure 31. Sequence 1 Call Value Plot](image1)

![Figure 32. Sequence 2 Call Value Plot](image2)

Analysis: In order to compare between the Binomial Option Pricing Model and the Black & Scholes results, an online calculator of the B&S model is used to output the results and conduct the difference. The B&S online calculator given the same input data resulted in the following [24] Call Price = $224.95 and Put Price = $88.34.

Investigating the three cases with respect to Black & Scholes Model:
- One period: the binomial call option value was equal to 255.047 which is 86.7% close to B&S model result.
- Two periods: the binomial call option value was equal to 200.495 which is 89.1% close to B&S model result.
- N sequence of periods: the binomial call option value converges to 220 which is 97.7% close to B&S model result.

5.2.6. Time Complexity and Parallel Computing

Using the binomial option pricing model on CAC 40 index data in the case of large number of periods, makes the program to run slower (time issue) and the R console does not input all the values of the tree (memory issue). Fortunately, R language provides a set of packages that allow
parallel processing such as the package “parallel” [25]. In fact, the model implies a loop and while increasing the number of periods the loop takes more time to generate the tree. As a result, the model’s time complexity converges to:

\[
\text{Time Complexity} = O(n^2)
\]

```
> system.time(BOMSeq(30))
[1] 30
  user  system elapsed
  0.001  0.000  0.001
> system.time(BOMSeq(100))
[1] 100
  user  system elapsed
  0.004  0.000  0.005
> system.time(BOMSeq(300))
[1] 300
  user  system elapsed
  0.046  0.002  0.052
> system.time(BOMSeq(800))
[1] 800
  user  system elapsed
  0.214  0.008  0.224
```

**Figure 33. Running time on R on Different Number of Periods**

Thanks to the multicore processors available currently, parallel processing is possible on the same computer. It requires to identify the number of cores the system has, which in this case is equal to 4, and disperse the work on them to better the program’s time efficiency.

Following is the program used to speed up the model in case of large number of sequential periods:

```r
#install.packages("parallel")
library(parallel)
library(whoR)
numCores <- detectCores()
cl <- makeCluster(numCores)
clusterEvalQ(cl, source("/Users/awc/Deskto/Spring2019/U/I/Capstone"))
periods = seq(100, 500)
periods = sample(periods)
x <- BOMSeq
clusterExport(cl, "x")
clusterEvalQ(cl, x)
```

**Figure 34. Parallel Computing on R**
6. STEEPLE Analysis

6.1 Social & cultural factor

This capstone project would have an impact socially and culturally for a set of reasons. Socially, investors would be able to use the binomial option pricing model to value their options. Culturally, and since the project applied the model on CAC 40 index data, it is going to be a good reference to analyze how healthy was the French market during 2018.

6.2 Technological factor

In terms of the technology, the project used the R programming language systematically. As a result, the project is an example of the efficiency such programming languages offer when applying financial models specifically and models in general.

6.3 Economic factor

This project deals with an in depth study of the binomial pricing model to predict CAC 40 index. In fact, the model is a very powerful tool and can be used by investors to better ensure their investments, hence pricing their options. Consequently, trading stocks will become more frequent which will help the market to grow.
6.4 Environmental factor
The project can be considered to not have bad impacts on the environment in any way possible.

6.5 Political factor
The project does not impact the politic of France nor the one of the other countries.

6.6 Legal & legislative factor
The project does not violate any laws or legislative statements; since the data used along with the formulas are all available on open sources.

6.7 Ethical factor
Throughout this project, the technical terms, data, along with the formulas have all been retrieved from reliable sources; which were all cited accurately to give the right credits to their authors.

6.8 Future Work
During this project, the binomial option pricing model was built gradually and was applied on CAC 40 index data. Thanks to the statistical programming language R a series of statistical analysis was conducted along with data visualization to better understand the index and value its options eventually. In fact, the performance of the model can be largely increased by improving the index volatility approximations. Besides, from time computing point of view, the model can be accelerated. In fact, According to a financial and quantitative analysis journal “the accelerated binomial can be used to value all the variety of options (on foreign exchange, commodities, futures, stocks that pay dividends, and so on) for which the binomial itself is applicable. In addition, it can be extended to price options whose value depends on two (or more) state variables.” [19]. Last but not least, the future work will be also centered around comparing the program’s performance with the one applied using the graphical processing units.
7. CONCLUSION

To conclude, this project was a detailed work on how to build the binomial option pricing model to predict CAC 40 index. In fact, a detailed background of option pricing was provided, along with definitions of the important technical terms to get an overall understanding of the model’s application. In addition to that, data visualization of the CAC 40 index was presented to better understand the French market’s state during 2018. As a result, the model proved to be very powerful given its relatively easy mathematical background, yet it gave enormous economic insights on the index along with option valuation. Consequently, the model can be used by investors to price options of different indices. Last but not least, the model proved to behave just like the Black and Scholes model in case of large number of periods with an approximation close to 98% which is definitely impressive given the model’s sophisticated simplicity.
8. REFERENCES


REVISITED. REVSTAT – Statistical Journal, [online] 9. Available at:


9. APPENDIX A

In this section, a set of screenshots are presented to give an idea of the main steps used to apply the model using the R programming language.

![Figure 36. CAC 40 Index Data Frame on R](image-url)
Figure 37. CAC 40 Index Symbol on R Environment

```r
rm(list=ls()) # this clears the global environment
set.seed(19940903)
setwd("C:\Desktop\Spring2019AU1\Capstone\capstoneData")
#install.packages("quantmod")
library(quantmod)
getSymbols("FCHI", from="2018-01-01", to="2019-01-01")
dfCAC40 <- as.data.frame(FCHI)
```

Figure 38. CAC 40 Index Descriptive Statistic on R

```r
# descriptive statistics
#install.packages("pastecs")
library(pastecs)
DescriptiveCAC40 <- stat.desc(dfCAC40)
DescriptiveCAC40 <- round(DescriptiveCAC40, 2)
# Graphical Analysis of CAC40
#install.packages("ggplyr")
#install.packages("ggplot2")
#install.packages("magentar")
library(ggplot2)
library(ggpubr)
# Histograms
gghistogram(dfCAC40$FCHI.High, xlab = "Index High", add = "mean")
gghistogram(dfCAC40$FCHI.Close, xlab = "Index Adjusted", add = "mean")
gghistogram(dfCAC40$FCHI.Volume, xlab = "Index Volume", add = "mean")
ggplot(dfCAC40$FCHI.Close, xlab = "Index Closing Value", color = "blue")
ggplot(dfCAC40$FCHI.Volume, xlab = "Index Volume Value", color = "darkblue")
# BAR CHART OF ONE YEAR: CAC40 INDEX
barChart(dfCAC40, theme= "white")
addMACD() # Moving Average Convergence Divergence (MACD)
addBBands() # Bollinger Band
addRSI()
chartSeries(dfCAC40, subset='last 3 months', theme = 'white')
```
Figure 39. CAC 40 Stock Tree

```c
85 - # Building a stock tree for BOPM
86 - StockTree = function(S, Sigma, DeltaT, N) {
87 -   tree = matrix(0, nrow=N+1, ncol=N+1)
88 -   # Calculate the value by which the stock can go up OR down (Geometric Brownian Motion)
89 -   u = exp(Sigma * sqrt(DeltaT))
90 -   d = exp(-Sigma * sqrt(DeltaT))
91 -   # Loop on the number of period simulated
92 -   for (i in 1:(N+1)) {
93 -     for (j in 1:i) {
94 -       tree[i,j] = S * u^((i-1) - d^((i-1) - (j-1)))
95 -     }
96 -   }
97 -   return(tree)
98 - }

100 - # Stock prediction tree in 2 periods
101 - treeCAC40 = StockTree(S=CAC40Adjusted[1], Sigma= CAC40Volatility2[1], DeltaT=1/2, N=2)
102 - X = treeCAC40[1]
103 - callOptionValue = max(c-X, 0)
104 - putOptionValue = max(X-c, 0)
```

Figure 40. CAC 40 Binomial Option Pricing Model

```c
117 - # BOPM which includes a generic function that outputs a list of results------------------------
118 - setwd("C:/Users/username/Projects/CAPM/Finance")
119 - alpha = alphaProbability/2, r = r, DeltaT = T/N, Sigma = Sigma)
120 - option = BOPM(CAC40tree, Sigma, DeltaT = T/N, r = r, X = X, type = type)
121 - return(list(CAC40_Stocks = CAC40tree, CAC40_Option = option, Price = option[1],
122 -   alpha = alpha, delta = delta))
123 - }
124 - }
125 - }
126 - # ONE PERIOD
127 - BOPM(type = 'call', Sigma = CAC40Volatility2[1], T = 1, r = 0.02, X = treeCAC40[1], S = treeCAC40[1], N = 1 )
128 - BOPM(type = 'put', Sigma = CAC40Volatility2[1], T = 1, r = 0.02, X = treeCAC40[1], S = treeCAC40[1], N = 1 )
129 - # TWO PERIODS
130 - BOPM(type = 'call', Sigma = CAC40Volatility2[1], T = 1, r = 0.02, X = treeCAC40[1], S = treeCAC40[1], N = 2 )
131 - BOPM(type = 'put', Sigma = CAC40Volatility2[1], T = 1, r = 0.02, X = treeCAC40[1], S = treeCAC40[1], N = 2 )
132 - # SEQUENCE OF PERIODS
133 - periods = seq(1, 30)
134 - BOPMee = function(period) {
135 -   print(period)
136 -   option = BOPM(type = 'call',Sigma = CAC40Volatility2[1],T = 1, r = 0.02, X = treeCAC40[1], S = treeCAC40[1], N =period)
137 -   return(option[Price])
138 - }
```