PORTFOLIO OPTIMIZATION MODEL IN THE MOROCCAN STOCK MARKET

FINAL REPORT

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PORTFOLIO OPTIMIZATION IN THE MOROCCAN STOCK MARKET

Capstone Report

Student Statement:
This project was designed to reflect my personal work and research under the supervision of Dr. Lahcen Laayouni. Throughout the design process, both public safety and ethics were taken into accounts, respected, and none of them was violated.

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Approved by the Supervisor

Dr. L. Laayouni
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ABSTRACT
This capstone project has as an objective the portfolio optimization based on Markowitz mean-variance model. The model consists of using Lagrange multipliers in order to optimize portfolio investments within the Moroccan stock market, by minimizing the portfolio’s risk for a certain expected return value. This result will then be assessed by testing the significance between the return of the optimal portfolio with the return of the same portfolio after seven days period, and the same for the risk. The results were that 95% of the times the expected return at time of investment is larger than the return after seven days, and that 95% of the time the risk of the same portfolio after seven days will be larger than the optimized one. Moreover, the project’s STEEPLE analysis will be introduced to assess its added value along with discussing the future work to be developed.

Keyword: portfolio optimization, Lagrange multipliers, STEEPLE, Morocco;
RESUME

Ce projet de synthèse a pour objectif l'optimisation du portefeuille sur la base du modèle de variance moyenne de Markowitz. Le modèle consiste à utiliser des multiplicateurs de Lagrange afin d'optimiser les investissements de portefeuille sur le marché boursier Marocain, et cela par minimiser le risque du portefeuille pour une certaine valeur de rendement. Ce résultat sera ensuite évalué en testant la signification de la différence entre le rendement du portefeuille optimal et le rendement du même portefeuille après sept jours, et la même chose pour le risque du portefeuille. Les résultats ont montré que 95% des fois le rendement attendu au moment de l'investissement est supérieur au rendement après sept jours, et que 95% du temps le risque du même portefeuille après sept jours sera plus grand que le risque optimisé. De plus, l'analyse STEEPLE du projet sera introduite pour évaluer sa valeur ajoutée, et les travaux futurs pour le développement du projet seront discutés.

Mot-clés: portefeuille, optimisation, Lagrange, STEEPLE, Maroc;
INRODUCTION

Investors in stock markets differ in their investment strategies based on personal or general constraints. However, all investors agree on the difficulty of assets’ allocation that will provide them with the best outcome, along with taking their preferences into account. The main reason behind this difficulty is the markets’ constant change that makes key variables like expected returns and risk hard to capture.

For this reason, many mathematicians and analysts developed throughout the years many mathematical models that can best answer investors’ questions about assets’ allocation. It is essential to note that models are mostly based on estimated variables, which leads to outcomes’ errors. Therefore, it is of great importance for investors to take into account these errors while making decisions.

1.1. Background

A financial portfolio can be defined as a set of assets that may be of different kinds. The main objective is to allocate the investor’s wealth in such a way that this set of assets (portfolio) is mostly profitable, along with taking into consideration the investor’s preferences (constraints). This objective can be achieved through portfolio optimization.

The roots of the portfolio optimization topic are closely linked to the portfolio theory. The latter was first introduced by Harry M. Markowitz during the 1950s, which is considered as “the father of modern portfolio theory” [1, Page 5]. The new concept in his theory is to focus on the investor and his or her investment preferences, rather than the market or other consumers. Within this focus, Markowitz argues that when an investor is willing to invest in securities with known future return, he or she will rationally pick the one with the highest return [2, Page 469]. However, this scenario is based on an unrealistic assumption, which is that future returns are known in advance.

The above stated assumption is very unlikely to happen in reality since markets’ behavior can be forecasted but within a range of uncertainty. Therefore, Markowitz suggested taking into
account while optimizing a portfolio, and in parallel with future excepted return, the risk of assets [2, Page 470]. The method of capturing the portfolio’s risk was based on its variance that includes the covariance that exists between different securities. Further research in the 1960s was done by “Sharpe, Blume, King, and Rosenberg [who] greatly clarified the problem of estimating covariance” [2, Page 470].

Now that the portfolio’s risk has become an important factor when investing, more attention was given to diversification. That was mainly due to the fact that portfolio’s risk is closely related to individual securities’ risk, along with the potential relationship that might exist between them. Within this framework, Bernoulli has stated in his 1738 article that “it is advisable to divide goods which are exposed to some small danger into several portions rather than to risk them all together” [3, Page 1041].

The importance of diversification emphasizes the idea that whether to hold an asset or not, an investor should not only base his or her decision on this assets’ return and risk, but should also include in the analysis the potential dependence of this asset to others included in the same portfolio. This perception was lacking in the research done by Williams in 1938 [4] along with Graham and Dodd in their 1934 published article [5].

Therefore, the question that has to be answered is how a portfolio can be diversified in order to achieve the desired outcomes, which what Markowitz and others later tried to answer through the mean-variance model.

The mean-variance models started to be widely used in optimizing portfolio until nowadays. These models are based on the concept of either maximizing the expected portfolio return for a predetermined variance, which represents risk, or minimizing the portfolio’s risk based on a target expected return [6, Page 49].

The model sat by Markowitz and the one by Roy in 1952 [7] were similar in terms of the variables used. They both tried to capture the portfolio’s risk using the portfolio’s variance, along with using the expected return as a sat constraint by the user. However, their model differs in terms of outcome. Markowitz’s model results in a set of portfolios called efficient
portfolios that construct the later discussed efficient frontier. The choice of which portfolio to choose is left to the investor. On the other hand, Roy is not giving the investor this choice, and his model always suggests the portfolio with the minimum risk globally.

The different models were constantly subjected to review and criticism, especially when it comes to its robustness and on how well the estimated values reflect the market’s reality. In addition, Markowitz didn’t stop his research on portfolio theory, and published in 1959 a book on assets allocations in financial portfolios [3, Page 1042]. The latter included more details about his model in order to serve the readers with limited quantitative background.

1.2. Capstone Project Aim
Within the framework of the capstone project, the latter will tackle the portfolio optimization topic in the Casablanca Stock Exchange in Morocco, later referred to as CSE. The goal is to apply a mean-variance model for a potential investment in CSE. The data was collected, and the model was implemented and assessed later for accuracy.

In addition, historical data analysis was used in order to provide the model’s users with potential scenarios after using the model. It is important to note that the model is mostly addressed to analysts that can use it as one of the tools when advising investors, since the data used should be subjected to selection before use.

1.3. Report Outline
To better present the project, the first chapter broadly tackled the presentation of the topic. The next chapter will introduce the methodology followed throughout the project. At first, the main variables that were used in the model will be defined, along with any related assumptions. Then within the same chapter, the mean-variance model used will be detailed.

The following chapters will discuss each the empirical results. The first implementation will be concerned with the mean-variance model implementation, and will discuss its findings.
The next chapter will tackle the model’s assessment method and its corresponding results. The chapter that follows will be about scenario generation based on historical data analysis.

Furthermore, a chapter about the STEEPLE analysis will be included in order to discuss the project’s impact from different perspectives. The succeeding chapter will tackle future works. The latter chapter will reflect on the project as a whole, and will present the further work that might be accomplished within this context. Finally, the conclusion will sum up the main findings of the project, and will emphasis on the most important project development that may be achieved later.
CHAPTER 1: METHODOLOGY

1.1. Introducing concepts

1.1.1 Expected Return

In the implemented model, one of the key variables is the assets’ expected return. As stated before, this return cannot be known in a certain way, which results to estimating it.

First, let’s assume a sample \( n \) of assets to be considered, who are indexed by \( i = 1, 2, \ldots, n \). Each asset will then have an expected return that will be referred to as \( r_i \). For this project, the expected return of each stock was estimated by the arithmetic mean of its rate of return, taking into consideration a sample of 250 days. So the vector \( r = (r_1, r_2, \ldots, r_n) \) will denote the expected return of each stock \( i \). Assuming normal distribution for this large sample, the mean can be considered as a good estimator for the expected value.

For the overall portfolio, its expected return, denoted \( r_p \), can be represented by the weighted sum of the considered assets:

\[
r_p = \sum_{i=1}^{n} w_i \cdot r_i \text{ with } w_i \text{ being the weight of asset } i
\]

The weight \( w_i \) represents the fraction of the overall budget that will be allocated to the asset \( i \). The objective of the project’s model is to determine the vector \( w = (w_1, w_2, \ldots, w_n) \) of individual allocations in order to attain the investor’s objectives.

The equation can be further simplified to the expression:

\[
r_p = w^t r
\]
1.1.2 Risk

The risk of a portfolio is related to the uncertainty of stocks’ returns. Depending on how it is combined, a portfolio’s risk can be estimated by the square root of the variance of its expected returns. This assumption was first introduced by Markowitz, and later on approved by many scholars. The remarkable work of Markowitz has provided evidence that investors will be more interested in the way a specific asset will contribute in the portfolio’s overall risk, rather the individual risk of that asset [3, Page 1042]. This means that the portfolio’s variance is combining both the risk of investing in a specific stock and the one coming from the potential relationships between stocks.

This concept will be clearer after developing the portfolio’s variance, where the covariance of each asset with another asset considered in the same portfolio will represent a significant term in the equation.

The variance of the portfolio can be represented by:

\[
Var_p = var(r_p) = var(w^t r) = \sum_{i=1}^{n} \sum_{j=1}^{n} Cov(w_i * r_i, w_j * r_j) \\
= \sum_{i=1}^{n} \sum_{j=1}^{n} (w_i \bar{r}_i w_j \bar{r}_j - w_i \bar{r}_i * w_j \bar{r}_j) = \sum_{i=1}^{n} \sum_{j=1}^{n} (w_i w_j * \bar{r}_i \bar{r}_j - w_i \bar{r}_i * w_j \bar{r}_j) \\
= \sum_{i=1}^{n} \sum_{j=1}^{n} w_i \bar{r}_i * Cov(r_i, r_j) * w_j
\]

This leads to the following equation:

\[
Var_p = w^T * COV * w
\]

With COV being the covariance matrix \(\text{COV} = \begin{bmatrix} \text{Cov}(r_1, r_1) & \cdots & \text{Cov}(r_1, r_n) \\ \vdots & \ddots & \vdots \\ \text{Cov}(r_n, r_1) & \cdots & \text{Cov}(r_n, r_n) \end{bmatrix}\)

To better understand the effect that a potential correlation between two stocks might affect the portfolio’s risk, we will consider an example of two assets. The overall portfolio variance in this case will be:

\[
Var_p = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1w_2\text{Cov}(r_1, r_2)
\]

6
Which later gives:
\[
Var_p = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1w_2\sigma_1\sigma_2 \rho(r_1, r_2)
\]
With \(\rho(r_1, r_2)\) being the correlation between \(r_1\) and \(r_2\)

So let us now discuss the extreme possible cases.

- **Case \(\rho(r_1, r_2) = +1\):** In this case, the variance of the portfolio will be at its maximum since all other terms are strictly positive.
- **Case \(\rho(r_1, r_2) = -1\):** In this case, the variance of the portfolio will be at its minimum since all the other terms will stay positive.

From the above mentioned cases, it is evident now that the correlation between two assets will impact the portfolio’s risk. Thus, the objective for any investor is to select the assets that are more probable to be negatively correlated with the objective of minimizing the variance, which will later minimize the portfolio’s risk.

### 1.1.3 Efficient Frontier

The efficient frontier refers to the “set of efficient mean-variance combinations” [1, Page 5]. What is meant by this Markowitz’s definition is that the efficient frontier lists all the optimal combinations of portfolios that are minimizing risk for a given expected return, or that are maximizing return for a given risk. The objective of the mean-variance model introduced by Markowitz was to build the efficient frontier taking into consideration a set of assets, and to let then the investor choose which efficient portfolio to adopt according to his or her investing strategy.

### 1.1.4 Short Sell

In the model that will be introduced, short selling will be allowed. By doing so, the weights of assets’ allocation will be allowed to have a negative value. It is important to define this term since the reader might not be familiar with it.

Short selling means the action of selling shares that have been borrowed from their owner for the purpose of making profit. The latter can be achieved by the hope of a share price decrease in the moment when the shares are being bought to be returned to the owner. In other words,
the investor estimates that a specific share that he or she doesn’t own is overestimated. Therefore, to quickly sell these shares before a decrease in their price, the investor borrows them and makes the deal.

However, these shares have to be returned to their owner, so at this time, the investor hopes that his or her expectations were right, and that the share price has been adjusted in the market. If it is the case, then this investor will be buying the shares at their fair value which is less than the price that was already established, and will return the shares to their owners. In this way, the investor will be making a profit that results from the difference of the prices.

1.2. Model Setting

The following is the mean-variance model that has been set for optimizing a portfolio of known expected return, which has been set by the investor.

**Objective**

The objective is to minimize the portfolio’s risk, which will imply minimizing the portfolio’s variance. The expression of the latter has been developed in the previous section, giving:

\[
\text{Min. } Var_p = w^T * COV * w
\]

**Constraints**

The model used is subject to two main constraints:

- The fixed budget by the investor that leads to \( \sum_{i=1}^{n} w_i = 1 \)
- The fixed expected return by the investor that gives \( \sum_{i=1}^{n} w_i r_i = r_p \). Which can be expressed as \( w^T * r = r_p \), where \( r_p \) is the required expected return by the investor, and \( r \) is the vector of stocks’ expected returns.

Note that the investment weights were not constrained to be positive, which will allow the investor to short sell in case of a negative value.
For this capstone project, only two constraints were considered. The reason behind it is that additional constraints will reflect investors’ preferences, whereas the project aims to discuss the general case scenario. Besides, it is explained in details how the model constraints are implemented in the solution; thus, only few modifications will be necessary in order to account for the new constraints. This topic will be further discussed in the future works’ chapter.

Solution

In order to solve this model, the method of Lagrange multipliers was used. We can define the following Lagrangian function with multipliers $u_1$ and $u_2$:

$$L(w, u_1, u_2) = w^T COV w - u_1 (w^T v - 1) - u_2 (w^T r - r_p)$$

Note that $v$ is defined as $v=[1,1,\ldots,1]$ where 1 is repeated $n$ times, $n$ being the number of stocks considered. In addition, each Lagrange multiplier is accounting for a specific constraint. Thus, having more constraints will lead to using a higher number of multipliers to account for them.

In order to obtain the critical points of this Lagrangian function, the following system must be solved:

$$\nabla_w L(w, u_1, u_2) = 2COV * w - u_1 * v - u_2 * r = 0$$

$$\frac{\partial L(w, u_1, u_2)}{\partial u_1} = w^T v - 1 = 0$$

$$\frac{\partial L(w, u_1, u_2)}{\partial u_2} = w^T r - r_p = 0$$

Note that the gradient’s use in the first equation is related to the fact that the function has been derived with respect to each term $w_i$.

To not overwhelm the reader with the detailed algebra, the system’s mathematical development is represented in Appendix B. The resultant system can be expressed as:

$$\begin{bmatrix}
v^T COV^{-1}v & r^T COV^{-1}v \\
r^T COV^{-1}v & r^T COV^{-1}r
\end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ r_p \end{bmatrix}$$
That can also be expressed as:

\[ A \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = 2 \cdot \begin{bmatrix} 1 \\ r_p \end{bmatrix} \]

with

\[ A = \begin{bmatrix} v^T \text{COV}^{-1} v & r^T \text{COV}^{-1} v \\ r^T \text{COV}^{-1} v & r^T \text{COV}^{-1} r \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \]

The previous system will have a unique solution when \( A \) is invertible, meaning that the determinant of \( A \) is different than zero. Then, the solution can be expressed as:

\[ \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \frac{1}{\det(A)} \cdot \begin{bmatrix} r^T \text{COV}^{-1} r & -r^T \text{COV}^{-1} v \\ -r^T \text{COV}^{-1} v & v^T \text{COV}^{-1} v \end{bmatrix} \cdot 2 \cdot \begin{bmatrix} 1 \\ r_p \end{bmatrix} \]

This results in the following solution:

\[ \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \frac{2}{\det(A)} \cdot \begin{bmatrix} r^T \text{COV}^{-1} r - r^T \text{COV}^{-1} v \cdot r_p \\ -r^T \text{COV}^{-1} v + v^T \text{COV}^{-1} v \cdot r_p \end{bmatrix} \]

At this stage, the value of the Lagrange multipliers has been determined; thus, the vector \( w \) is the only unknown left in the first system. The vector \( w \) will be determined by solving the following equation:

\[
2 \text{COV} \cdot w - u_1 \cdot v - u_2 \cdot r = 0
\]

\[
\equiv 2 \text{COV} \cdot w - \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \cdot \begin{bmatrix} v \\ r \end{bmatrix} = 0
\]

\[
\equiv w = \frac{1}{2} \text{COV}^{-1} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \cdot \begin{bmatrix} v \\ r \end{bmatrix}
\]

Then the optimal weights’ vector can be further developed as:

\[
w = \frac{1}{2} \text{COV}^{-1} \cdot \frac{2}{\det(A)} \cdot \begin{bmatrix} r^T \text{COV}^{-1} r - r^T \text{COV}^{-1} v \cdot r_p \\ -r^T \text{COV}^{-1} v + v^T \text{COV}^{-1} v \cdot r_p \end{bmatrix} \cdot \begin{bmatrix} v \\ r \end{bmatrix}
\]

\[
\equiv w = \frac{\text{COV}^{-1}}{\det(A)} \begin{bmatrix} v(r^T \text{COV}^{-1} r - r^T \text{COV}^{-1} v \cdot r_p) \\ r(-r^T \text{COV}^{-1} v + v^T \text{COV}^{-1} v \cdot r_p) \end{bmatrix}
\]
CHAPTER 2 EMPIRICAL RESULTS

2.1 Model Implementation

At this stage, the model was developed and explained in the previous chapter. The current objective is to implement it using a software in order to be able to comment on its results. For this project, the software that will be used is R for its convenience when dealing with stocks.

The implementation part was conducted for three different samples. In the first case, three listed companies in the CSE were considered randomly from three different industries. For the second case, 37 active companies in the CSE were considered for implementation. The third case was based on a more realistic scenario, and accounted for diversified and active assets.

2.1.1 Case1: Three Diversified Assets

The purpose of this first case is to be familiarized with the model, and to rightly code it in R. The three companies that were included are BMCE, COSUMAR, and ADDOHA. These companies are operating in three different sectors, and they were purposely chosen in this way for diversification.

The first step was to collect the data. In order to do so, the daily stock price of 250 days of each asset was recorded from the Thomson Reuters Eikon’s database. The next step was to code the previously developed model in R and run it for the given data. The code that has been used for this case is reported in Appendix C, with the running results.

The results show that for the specified expected return of 0.001 by the investor, the latter should short sell 26.23% of wealth in ADDOHA stocks, and invest 69.96% of the overall budget in BMCE along with 56.27% in COSUMAR. The risk of this portfolio is approximately 1.18%.

The efficient frontier was also plotted in order to visualize the behavior of the efficient portfolios that can be combined using these three assets. If an investor is not sure about his or
her preferred return, the efficient frontier plot represents a useful tool that regroups all optimal solutions in one graph.

![Efficient Frontier of Three Assets](image)

**Figure 1. Efficient Frontier of Three Assets**

### 2.1.2 Case 2: Active Assets

For this case, the assets that have been included are the active stocks in CSE. As a criterion of activity, the assets that are included in the MADEX index were included in the portfolio allocation sample. The MADEX index is an index that accounts for the most active Moroccan shares, and it is currently composed of 47 companies.

However, not all 47 companies were included in the model sample. The reason behind it is that some of the companies have been in the market for not long enough to have sufficient historical data for the analysis. Moreover, some other companies are not reporting their stock price in a daily basis. This will cause a problem for the model since in the risk computation depends on the covariance matrix, which is based on the daily price relationship between stocks. Therefore, these companies were also discarded. The final list of the considered companies is included in Appendix D.

Then, the necessary data was collected through the Thomson Reuters Eikon’s database, which consists of the 250 daily stock prices of all these companies. Following this step, few modifications were made to the previous R code in order to account for the differences in cases,. This code is represented in Appendix E.
The resultant weights will lead to a risk of 0.303% for a given return of 0.001. The efficient frontier of inventing in these 37 companies is illustrated by the following graph.

![Graph showing efficient frontier](image)

**Figure 2. Efficient Frontier of the 37 Active Assets**

### 2.1.3 Case 3: Diversified and Active Assets

For this third case, the asset’s selection will be based on two criteria. The first one is to be among the active companies in the CSE, which is the list in Appendix D. Using again the latter, a new criterion is introduced, which is the P/E ratio of the companies. This ratio is computed by dividing the market share price by the earning per share. This ratio can be interpreted as being the amount of money needed to raise one money unit of earnings.

On this basis, the sample of this case will be composed of the active assets representing each a single industry. To choose which company to consider in the portfolio, the company with the lowest P/E ratio of each industry will be chosen. In addition, if only one company is representing a certain industry, then this company will automatically be included.

The reason behind introducing this third case is for attempt to produce a more realistic scenario. That is because if an investor is willing to use this model, he or she needs to first choose which assets are to be included. Since a portfolio cannot in reality include investments in 37 different assets, adding a new criterion will help investors to narrow their selection.

The assets that were included in this portfolio are listed in Appendix F. Then, the same code was ran for a different value of N that represents the number of assets considered while using the same data collected previously.
The results of this efficient portfolio report a risk of 0.415% for a given return of 0.001. The efficient frontier of investing in these 20 companies is illustrated by the following plot:

![Efficient Frontier of the 20 Diversified Active Assets](image)

**Figure 3. Efficient Frontier of the 20 Diversified Active Assets**

**2.1.4 Discussion**

From the three cases above, we can use the efficient frontier plots to better understand a portfolio’s behavior. When comparing the three plots, it is evident that the more portfolios are included, the less risky the portfolio will be.

In addition, when taking the sample of 20 assets from the 37 according to the P/E criterion, the efficient portfolio’s risk was actually higher than the previous one. When taking this 20 assets’ sample the idea was to eliminate any high correlation that might be present due to the operation within the same industry. Eliminating this potential positive correlation would have resulted in a lower risk. However, this is opposite to what was found when running the model. Therefore, it might be the case that other factors are affecting the risk in a more important manner than this diversification, or that the criterion used of P/E ratio is not sufficient.

**2.2 Model Assessment**

The objective of this section is to assess how reliable can the implemented model be. Any investor that will follow the model’s result would like to know how the expected return and risk will differ from the real values, in case the portfolio weights were maintained.

In order to attain this objective, the optimal portfolio was compared to the one of similar weights but after a certain period. We chose a short period of seven days. The purpose is to test whether both the return difference and the risk difference are significant.
In order to do so, necessary data was collected. Since the sample of 37 assets has performed better in terms of risk, the same sample will be chose but with including more historical data. The idea is to run the optimization model as much as possible so that the sample of the return and risk differences will be large enough.

In order to collect this sample, the code in Appendix G was implemented. The resultant differences of returns and risk are represented by their density functions as follows:

![Density Function of the Returns Difference](image1)

**Figure 4. Density Function of the Returns Difference**

![Density Function of the Risks Difference](image2)

**Figure 5. Density Function of the Risks Difference**

The above densities can be approximated to the normal distribution, then the following hypotheses can be tested:

\[ H_0: \mu = 0 \]

\[ H_1: \mu \neq 0 \]
2.2.1 Returns difference testing

In order to test the above stated hypothesis, we will run a t-test taking the variable $r_{diff}$ as having a normal distribution, while $r_{diff}$ represents the difference of the expected return sat by the investor in the beginning of the portfolio’s investment, and the actual return made with the same portfolio after seven days. The following command was run in R:

```R
> t.test(rdiff, mu=0)

One Sample t-test

data: rdiff
t = -4.3073, df = 127, p-value = 3.278e-05
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
-1.534286e-04 -5.683257e-05
sample estimates:
mean of x
-0.0001051306
```

The p-value is very small, so the null hypothesis will be rejected. This means that 95% of the times the difference in the returns is significant and not equal zero.

Now the interest would be to test the following hypothesis:

\[
H_0: \mu_1 = \mu_2 \\
H_1: \mu_1 > \mu_2
\]

With $\mu_2$ being the mean returns after the seven days period, and $\mu_1$ the mean returns at time of investment.

After running the following commands in R, we get the result:

```R
> t.test(returns[,1], returns[,2], alternative="greater", paired=TRUE , var.equal=FALSE)

Paired t-test

data: returns[, 1] and returns[, 2]
t = 4.3073, df = 127, p-value = 1.639e-05
alternative hypothesis: true difference in means is greater than 0
95 percent confidence interval:
6.468883e-05 Inf
sample estimates:
mean of the differences
0.0001051306
```
The very low value of p-value affirms that the null-hypothesis can be rejected. This means that 95\% of the times the mean returns after the seven days period is less than the mean returns at time of investment.

### 2.2.2 Risk difference testing

Same as the $r_{diff}$ variable, the hypothesis was tested for the $risk_{diff}$ variable, which represents the difference of the minimized risk by the model and the actual risk made with the same portfolio after seven day. We assume that the variable is following a normal distribution too. In order to do so, the same previous commands were used in R:

```r
> t.test(riskdiff, mu=0)
```

```
One Sample t-test

data:  riskdiff
t = 4.9672, df = 127, p-value = 2.145e-06
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:  
9.260285e-05 2.152389e-04
sample estimates:
  mean of x
0.0001539209
```

As it was the case for returns, the risk difference is significantly different than zero since the p-value is very low. This implies that 95\% of the times the difference in the risk is to be considered as significant.

Now the interest would be to also test for the following hypothesis:

\[
H_0: \mu_1 = \mu_2
\]

\[
H_1: \mu_1 < \mu_2
\]

With $\mu_2$ being the mean risk after the seven days period, and $\mu_1$ the mean risk at time of investment.
After running the following commands in R, we get the result:

```r
> t.test(risks[,1], risks[,2], alternative="less", paired=TRUE, var.equal=FALSE)

Paired t-test

data:  risks[, 1] and risks[, 2]
t = -4.9672, df = 127, p-value = 1.072e-06
alternative hypothesis: true difference in means is less than 0
95 percent confidence interval:
  -Inf -0.000102577
sample estimates:
  mean of the differences
    -0.0001539209
```

The very small value of p-value affirms that the null-hypothesis can be rejected. This means that 95% of the times the mean risk after the seven days period is higher than the mean risk at time of investment.

### 2.2.3 Discussion

The results of the hypotheses testing should be discussed. The fact that 95% of the times the mean returns after the seven days period is less than the mean returns at time of investment implies an interesting outcome. Since the expected return was first determined by the investor, the latter can use the test’s result to check when the optimization model should be rerun. This means that the investor should make sure to reinvest in different weights whenever the achieved return is significantly less of what is desired.

For the risk hypothesis testing, its result represents important information regarding the model. Since 95% of the times the mean risk after the seven days period is higher than the mean risk at time of investment, the model can be considered as no more efficient after a period of seven days.

However, the tests’ results are not to be considered as definite. That is because these results might change in case of changing the return specified by the investor. To elaborate more, the two samples that were tested are directly linked to the return that was first specified by the investor, and also by the maximum return that we sat in order to generate the efficient frontier.
CHAPTER 3: STEEPLE ANALYSIS

The following chapter will tackle the STEEPLE analysis. The latter is concerned with discussing the societal, technical, environmental, ethical, political, legal, and economic aspects of the capstone project. Moreover, the future work that can be implemented later in the project will be discussed.

3.1. STEEPLE analysis

Societal
In terms of society, this project will help investors to optimize their portfolios along with investing managers to optimize the ones of their clients. Perhaps the model will not provide the ultimate portfolio combination, but it will provide a great guidance regarding the trend that a certain portfolio combination might have.

Technical
From the technical viewpoint, the project only uses the statistical software R. The latter represents a convenient way for the model implementation but it demands a prior knowledge by any model user.

Therefore, the model can be more technically developed to ease the usage by people with no coding background.

Environmental
The implementation of the model will not affect the environment in an important manner.

Ethical
All the information that has been used are publically stated, and represent common information. Moreover, any model that has been used to build the final model was attributed to its author. In addition, the investors that may use the model are aware of the possible uncertainty, the possible risk, and to what extent the model can be reliable.
Political
The project doesn’t affect the country’s politics. However, if any law that has been passed by the government and that affect investing regulations should definitely be included in the model. These new regulations can be taken into account by adding new constraints to the model.

Legal
The model doesn’t break any law, and any kind of illegal acts like insider trading are not taken into account while modelling.

Economic
This project can impact the economy in a positive way. That is because when investors are more confident about the method used when investing, and the fact that they can limit their risk will make them more motivated to invest their money. This will immediately affect the trading by increasing it, making stock prices to rise which will later result in a market growth.

3.2. Future work
Constraints
The discussed model can eventually include more constraints than the two implemented. The investor may add a constraint related to liquidity of the assets, or a constraint related to the profitability. Moreover, short selling is often regulated by strict regulations that can also be included as constraints in order to make to model more realistic.

Another important constraint might be the setting of boundaries that some specific weights should not exceed, depending on the industry or different criteria. An additional constraint may be related to the previously invested portfolio, which will set the percentage increase in return or decrease in risk necessary to change the current portfolio combination.

Diversity
The model can be further developed in order to include different securities’ types in the portfolio. An investor would like to diversify his or her portfolio not only within the same stock market, but perhaps to include other types of assets. The challenge in this case will be related to capturing the present risk within the overall portfolio.
Developed platform

Moreover, an important project that can be later developed can be related to making the process of optimizing a portfolio automatic. A software can itself collect data in a daily basis, and suggest a portfolio combination to the investor based on his or her constraints. Then the software will assess whether the new return or risk is better to the already implemented one, which will allow the investor to have a clear vision about whether to change the current investment or not.

The new implemented platform should be very easy to use, since it should take into account users with little or no statistical and technical background. Moreover, it should take into consideration the different constraints that should be sat by the user. An additional feature will be to give the user the choice to either minimize the risk for a specific return, or to maximize the return for a specific level of risk.
CONCLUSION

To sum up, the model that has been implemented can be considered as a useful tool that investors or others can use to understand the trend that a specific portfolio combination might have. The purpose of the model was to determine the necessary budget weights to be assigned to each considered asset, and that is with the objective of minimizing the overall portfolio risk for a specific amount of expected return.

Moreover, the model results have allowed the plotting of the efficient frontier, which includes the set of efficient portfolios for each level of return. These plots have helped in a better understanding of the effect of diversification, and that is by comparing the plots when investing in three assets, 20 assets, and 37 assets.

After testing the significance of the expected return as set by the investor and the same portfolio’s return after seven days, it is 95% of the times true that the expected return at time of investment is larger than the other return. The significance of risk difference was also tested, and the result was that 95% of the time the risk of the same portfolio after seven days was larger than the optimized ones. These results will help in the future development of the project, but it is important to note that these results can differ by changing the sample.
REFERENCES


APPENDIX A – Project Specifications

LYAMANI Mouna
EMS
STATISTICAL APPROACH TO PORTFOLIO OPTIMIZATION
LAAYOUNI L
Spring 2018

The objective of this capstone design project is to use statistical methods for portfolio optimization, which requires minimizing the portfolio risk for a fixed profit or maximizing this profit for a given risk.

In order to accomplish this task, the focus will be on the companies listed in the Casablanca Stock Exchange. The rate of return of the active companies along with their volatility will be taken into account when making the portfolio investments. As a start, the portfolio will be composed of four corporations. The objective is to find the weights that should be assigned to each investment while accounting for the overall risk. For this matter, the essential equations will be developed using algebra, and then the problem will be modelled using the software R. The latter will optimize either the rate of return, or the risk, while taking into account the different constraints.

The next step will be scenario generation. From the resultant weights, different scenarios should be interpreted for the given portfolio investments. Meaning that the results should be translated to common language so that any investor, regardless of their financial knowledge, would be able understand the model’s results.

A further possible step will be to program the whole process of optimization. The purpose is to code a full script that will include the different steps of the model. This will allow any user to input the potential companies’ rate of return, and either the target risk or profit, and then the program will generate the optimal investment.

This project will require first a good understanding of algebra, quantitative methods, and portfolio theory along with software skills such as R. Besides, the access to data is fundamental, so the stock prices of the listed companies need to be available. A possible resource can be the online website of the Casablanca stock exchange, or through the Thomson Reuters Eikon database.
APPENDIX B - Mathematical Development of the Lagrangian Equations

We start by the following system

\[ \nabla_w L(w, u_1, u_2) = 2COV \ast w - u_1 \ast v - u_2 \ast r = 0 \]
\[ \frac{\partial L(w, u_1, u_2)}{\partial u_1} = w^T \ast v - 1 = 0 \]
\[ \frac{\partial L(w, u_1, u_2)}{\partial u_2} = w^T \ast r - r_p = 0 \]

From the first equation, we get the following:

\[ w = \frac{1}{2} COV^{-1}(u_1 \ast v + u_2 \ast r) \]

And from the second one:

\[ w^T \ast v = 1 \]
\[ \equiv \left( \frac{1}{2} COV^{-1}(u_1 \ast v + u_2 \ast r) \right)^T \ast v = 1 \]
\[ \equiv v^T COV^{-1} v \ast u_1 + r^T COV^{-1} v \ast u_2 = 2 \]

And from the third equation:

\[ w^T \ast r = r_p \]
\[ \equiv \left( \frac{1}{2} COV^{-1}(u_1 \ast v + u_2 \ast r) \right)^T \ast r = r_p \]
\[ \equiv v^T COV^{-1} r \ast u_1 + r^T COV^{-1} r \ast u_2 = 2r_p \]

Combining the resultants of the second and third equations will result in the following system:

\[ \begin{bmatrix} v^T COV^{-1} v & r^T COV^{-1} v \\ r^T COV^{-1} v & r^T COV^{-1} r \end{bmatrix} \ast \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2r_p \end{bmatrix} \]
APPENDIX C - Model’s R Script Run for Three Assets

```r
> #Reading the data
> #Number of assets considered is N
> #Number of daily stocks considered is D
> D=250
> N=3
> data3<-read.delim(file.choose(),header=T)
>
> #Defining first the RoR function
> #Giving prices' vector, the function will return the ROR
> ror<-function(x){
+  l<-length(x)
+  y=c(rep(0,l-1))
+  for (i in 1:(l-1)) {
+    y[i]=(x[i+1]-x[i])/x[i]
+  }
+  return(y)
+ }

> #creating the RoR matrix
> ror3=matrix(nrow=(D-1),ncol=N)
> for (i in 1:N) {
+  ror3[,i]<-ror(data3[,i])
+ }

> #creating the expected return vector
> r<-c(rep(0,N))
> for (i in 1:N) {
+  r[i]<-mean(ror3[,i])
+ }

> #computing the covariance matrix
> Cov=cov(ror3,ror3)

> ##Lagrangian method
> #rp is portfolio return given by the investor
> rp=0.001
> w<-c(rep(0,N))
> CovInv<-solve(Cov)
> A=matrix(c(rep(0,4)),nrow=2)

> v=c(rep(1,N))
> a11<-A[1,1]<-v'*CovInv%*%v
> a22<-A[2,2]<-r'*CovInv%*%r
> b=c(2,2*rp)
> u<-solve(A,b)
> #the optimal weights
> w<-CovInv%*%(u[1]*v+u[2]*r)/2

> #corresponding portfolio risk
> riskp=0
> riskp<-t(w)%*%Cov%*%w
> sqrt(riskp)

> #Getting the frontier
> #rmax is the maximum return considered
> #M is the number of portfolio in the return
> rmax=0.01
> M=100
> rfr=c(rep(0,M))
> riskfr=c(rep(0,M))
> wfr<-matrix(c(rep(0,3*M)),nrow=3)
> ufr<-matrix(c(rep(0,2*M)),nrow=2)
> bfr<-matrix(c(rep(0,2*M)),nrow=2)
> for (m in 1:M) {
+  rfr[m]<-m*rmax/M
+  bfr[,m]<-c(2,2*rfr[m])
+  ufr[,m]<-solve(A,bfr[,m])
+  wfr[,m]<-CovInv%*%(ufr[1,m]*v+ufr[2,m]*r)/2
+  riskfr[m]<-sqrt(t(wfr[,m])%*%CovInv%*%wfr[,m])
+ }
> plot(riskfr,rfr)
```
## APPENDIX D - List of the Selected 37 Active Companies

<table>
<thead>
<tr>
<th>Company</th>
<th>P/E</th>
<th>Industry</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENNAKL</td>
<td>11.62</td>
<td>Auto Vehicles, Parts &amp; Service Retailers</td>
</tr>
<tr>
<td>ATTIJARIWAFAR BANK</td>
<td>17.40</td>
<td>Banks</td>
</tr>
<tr>
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<td>19.83</td>
<td>Banks</td>
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<td>Banks</td>
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<td>M2M GROUP</td>
<td>26.80</td>
<td>Business Support Services</td>
</tr>
<tr>
<td>SNEP</td>
<td>22.10</td>
<td>Commodity Chemicals</td>
</tr>
<tr>
<td>COLORADO</td>
<td>21.57</td>
<td>Commodity Chemicals</td>
</tr>
<tr>
<td>DISWAY</td>
<td>15.45</td>
<td>Computer Hardware</td>
</tr>
<tr>
<td>DELTA HOLDING</td>
<td>15.36</td>
<td>Construction &amp; Engineering</td>
</tr>
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<td>LAFARGEHOLOCMAROC</td>
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<td>Construction Materials</td>
</tr>
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<td>Construction Materials</td>
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</tr>
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<td>CARTIER SAADA</td>
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<tr>
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<td>23.83</td>
<td>Transportation</td>
</tr>
<tr>
<td>CTM</td>
<td>15.43</td>
<td>Transportation</td>
</tr>
</tbody>
</table>
APPENDIX E - Model’s R Script Run for 37 Assets

```r
# Reading the data
# Number of assets considered is N
# Number of daily stocks considered is D

D=250
N=37
data37<-
read.delim(file.choose(), header=T)

# Preparing the Data
# creating the RoR matrix
ror37=matrix(nrow=D-1,ncol=N)
for (i in 1:N) {
    ror37[,i]<-
    -ror(data37[,i])
}

# creating the expected return vector
r<-c(rep(0,N))
for (i in 1:N) {
    r[i]<-
    -mean(ror37[,i])
}

# computing the covariance matrix
Cov=cov(ror37,ror37)

# Lagrangian
# rp is portfolio return given by the investor
rp=0.001
v=c(rep(1,N))

w<-c(rep(0,N))
CovInv<-
solve(Cov)
A=matrix(c(rep(0,4)),nrow=2)
a11<-A[1,1]<-
-v*CovInv%*%v
a12<-a21<-A[1,2]<-
-A[2,1]<-
-r*CovInv%*%v
a22<-A[2,2]<-
-r*CovInv%*%r
b=c(2,2*rp)
u<-solve(A,b)

# the optimal weights
w<-CovInv%*%(u[1]*v+u[2]*r)/2
# corresponding portfolio risk
riskp<-varp<-
0
Varp<-t(w)*Cov%*%w
riskp<-sqrt(t(w)*Cov%*%w)

# Getting the frontier
rmax=0.01
M=100
rfr=c(rep(0,M))
riskfr=c(rep(0,M))

for (m in 1:M) {
    rfr[m]<-
    m*rmax/M
    bfr[,m]<-
c(2,2*rfr[m])
    ufr[,m]<-
solve(A,bfr[,m])
    wfr[,m]<-
    CovInv%*%(ufr[1,m]*v+ufr[2,m]*r)/2
    riskfr[m]<-
    sqrt(t(wfr[,m])*CovInv%*%wfr[,m])
}

plot(riskfr,rfr)
```

### APPENDIX F - List of the Selected 20 Diversified and Active Companies

<table>
<thead>
<tr>
<th>Company</th>
<th>P/E</th>
<th>Industry</th>
</tr>
</thead>
<tbody>
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<td>Oil &amp; Gas Refining and Marketing</td>
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<td>RES DAR SAADA</td>
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<td>Telecommunications Services</td>
</tr>
<tr>
<td>CTM</td>
<td>15.43</td>
<td>Transportation</td>
</tr>
</tbody>
</table>
# Reading full data
> N=37
> D=250
> rp=0.01
> datafull=read.delim(file.choose(),header=T)
>
> # Initiating the matrices that will contain the results of each optimization run
> w=matrix(nrow=N,ncol=128)
> returns=matrix(nrow=128,ncol=2)
> vars=matrix(nrow=128,ncol=2)
> risks=matrix(nrow=128,ncol=2)
> us=matrix(nrow=2,ncol=128)
> v=c(rep(1,N))

# Lagrangian Function
# rp is portfolio return given by the investor
> lagrangian=function(rp,Cov,v){
+   w=c(rep(0,N))
+   CovInv=solve(Cov)
+   A=matrix(c(rep(0,4)),nrow=2)
+   a11=A[1,1]=v%*%CovInv%*%v
+   a22=-A[2,2]=-r%*%CovInv%*%r
+   b=c(2,2*rp)
+   u=solve(A,b)
+   return(u)
+ }

> for (j in 1:128) {
+   # data for each sample
+   data=datafull[j:(j+249),1:N]
+   # optimization
+   # creating the RoR matrix
+   ROR=matrix(nrow=D-1,ncol=N)
+   for (i in 1:N) {
+     ROR[,i]=-ror(data[,i])
+   }
+   # creating the expected return vector
+   rafter=c(rep(0,N))
+   for (i in 1:N) {
+     rafter[i]=-mean(RORafter[,i])
+   }
+   # returns after 7 days
+   returns[j,2]=w[,j]*rafter
+   # variance after 7 days
+   vars[j,2]=t(w[,j])%*%Covafter%*%w[,j]
+   # risk after 7 days
+   risks[j,2]=sqrt(vars[j,2])
+   }
+ }

# We can get now the difference of return, variance, and risk after
> rdiff=-returns[,2]-returns[,1]
> vardiff=-vars[,2]-vars[,1]
> riskdiff=-risks[,2]-risks[,1]
+ }
> plot(density(rdiff))
> plot(density(vardiff))
> plot(density(riskdiff))