AN R IMPLEMENTATION TO FINANCIAL RISK MEASURING USING VALUE AT RISK

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1. Abstract

The following document is a detailed report of my capstone project entitled “AN R IMPLEMENTATION TO FINANCIAL RISK MEASURING USING VALUE AT RISK”. The work presented in this report is spread over different fields mainly Computer Science, Finance, and Mathematics. The report starts with an introduction that gives some historical background about risk management regulations, how they developed over the years, and how they led to the choice of the Value at Risk as an important measure in financial risk assessment. Then the methodology part includes five main chapters that are the core of this project. It starts with providing a definition of the Value at Risk with all the parameters that relate to it such as the confidence levels and the time horizon. Then, it introduces three main methods that are very important in implementing VaR calculations which are the Historical Simulation, the Linear/Quadratic Model, and the Monte Carlo Simulation which seems to be the most important one. This section covers implementation of both the Historical and the Monet Carlo Simulations in R statistical language with full and detailed descriptions. Interpretations and comparisons of the results methods are provided and findings of each confirm those of the other about the significance of diversification in financial risk management. The tools and technologies along with the packages and functions are also explained. A demo is provided at the end of this document in the appendix with all explanations and code snippets that would be helpful to better understand this project.
2. Introduction

Recently, modelling of financial risk has become a significant issue in the world of banking and finance. The focus on evaluating risk in the whole financial portfolio has increased as well. This interest in this topic is due to a great extent to the changes in the laws and regulations established in the financial & banking industry. By this, we refer to the Basel Committee Accords and Frameworks. It is a committee for Banking Rules and Supervisory Practices that was founded in 1974 by the Governors of the Central Bank. The Committee has its headquarter in Basel, Switzerland, at the Bank for International Settlements [1]. It aims at promoting financial stability by enhancing the quality of banking surveillance in the whole world and to act as a convention for consistent collaboration between its member countries on supervisory issues in banking. The BIS provides instructions to financial institutions such as banks regarding how to manage their capital. The guidance and recommendations given by the Bank for International Settlements are considered as best practices worldwide [1].

Due to the changes in the banking and finance field, the need for using risk measures in evaluating and managing credit portfolios has grown significantly. Of these risk methods, we mention the Value at Risk, namely VaR that is widely used by financial managers and analysts.

The VaR is risk method that uses statistical analysis of financial data to approximate how likely the loss in a particular portfolio will exceed some amount. There are several methods to calculate the value at risk of a portfolio which will be presented in the chapters of this report. They all have a large effect on the way a financial institution handles its portfolios [1].

The VaR measure was integrated in the Basel II Framework. It was established in June 1999 as a refined Accord to replace the one issued in 1988. It was improved to address risk issues other than the credit risk, which was very well outlined and detailed in the previous Accord [2]. The BIS Committee released the “Market Risk Amendment” in June 1996, right before delivering the Basel II, which was designed to include in the new Accord a major requirement
for the market risks emerging from exposure of banks to equities, traded securities, options, foreign exchange... The most important aspect that Market Risk Amendment was that it allowed financial institutions, for the first time, to make use of internal models which are the Value at Risk models as a fundamental for estimating their market risk capital requirements, which is exposed to tough qualitative and quantitative standards [3].

3. The VaR Measure Definition

Risk measures give important information about risks to the financial risk managers in a financial institution. The Value at Risk measure is a statistical method that computes a single number to summarize the overall risk in an assets’ financial portfolio. It could also be used in defining the capital that a bank is required to keep with respect to the risks it is taking [4].

When a financial manager uses Value at Risk, he or she is making the following statement:

“I am X percent sure the institution will not incur a loss of more than V dollars in the upcoming N days.”

The variable V denotes the VaR quantile of the portfolio. Value at Risk is a function with two parameters: the confidence level (X %) and the time horizon or the holding period.

VaR could also be defined as the loss level over a period of N days that has a (100-X) % probability of being exceeded. In banks, regulators require VaR calculation for market risks with a time horizon of 10 days and a confidence level of X = 99 %.

If we take N days as the holding period and X % as the confidence level, the value at risk is the loss that corresponds to (100-X) th percentile of the portfolio value gain distribution over the upcoming N days [4].

Note:

If we take into account the probability distribution of the gain, the value at risk is related to the left part of the distribution whereas if we consider the loss distribution, the value at risk is related to the distribution’s right tail. In the first case, the loss is a negative gain whereas in the second the gain is a negative loss [4].
Example:

If we take $N=5$ days and $X=97\%$, the value at risk is 3% of the gain’s distribution in the portfolio value over the upcoming 5 days. VaR is easy to understand and use which makes it a measure that is very interesting. Basically, it asks the following question that every financial manager wants to answer “How bad things can get?” Managers think that is fine to restrict all the Greek letters Alpha, Beta, Sigma...of all market variables determining a portfolio in one single quantile [4].

In case we believe that a portfolio risk could be described using the single amount obtained from the VaR, we should think whether it is the best choice or not. Researchers stress on the fact that VaR measure could induce traders to choose a portfolio with large potential losses as shown in Figure 21.2. The figures below show two portfolios with similar VaR, but the second one is much riskier [4].

Figure 1: VaR calculations from the probability distribution of portfolio value change

Figure 21.1 Calculation of VaR from the probability distribution of the change in the portfolio value; confidence level is $X\%$. Gains in portfolio value are positive; losses are negative.

Figure 21.2 Alternative situation to Figure 21.1. VaR is the same, but the potential loss is larger.
In order to solve the problem mentioned above, managers tend to use another measure called “Expected Shortfall”. It determines the expected loss over a period of N-days with the condition that an event in the (100-X) % left part of the distribution happens [4].

**Time Horizon/ Holding Period**

As mentioned previously, VaR measure accepts two parameters which are the confidence percentage and the time horizon that is given by \( N = \text{Number of days} \). Practically, managers initialize \( N=1 \) in the first time. The reason is that they are lacking data to provide a direct estimation of the market variables’ behavior over longer time periods. The following assumption is made most of the time [4]:

\[
N - \text{day VaR} = 1 - \text{day VaR} \times \sqrt{N}
\]

The above formula is exactly accurate only if the changes in the portfolio values that happen on consecutive days have independent similar normal distributions with an average of zero [4].

4. **Technology and Tools Used**

4.1. **R Programming Language**

R language is a statistical language used for a big variety of statistical computations such as building models whether they are linear or nonlinear, analyzing time series, implementing financial and quantitative analysis...It is easy to use and also allows creation of graphics thanks to the graphical tools that are highly wide-ranging. R is available on the website of the “r-project” under the GNU public license. On the website, there exists complete and comprehensive documentation [5].

4.2. **Packages Used**

4.2.1. **Quantmod**

Also called « Quantitative Financial Modelling Framework, Quantmod is a package that belongs to R’ comprehensive archive network is built to help financial managers assess, build
and specify the financial quantitative approaches. It allows the use of financial data that could be loaded from the internet and then manipulated using its functions. This package is also known as a fast environment for prototyping because it gives simple and straightforward functions that would provide the needed functionalities without requiring an extensive learning before use [6].

4.2.2. PerformanceAnalytics

This package that is also available in R archive network provides users with the necessary econometric tools and functions for assessing performance and analyzing risk in financial portfolios. It makes it easy for researchers and financial programmers, analysts, or managers to analyze returns streams whether they are normally or non-normally distributed (R studio). This package allows users to manipulate returns’ data and all the functions it provides could work with different kinds of periodicity whether it is monthly, daily or yearly. Besides, it uses data from time series which is the one I have worked with in my historical simulation and allows risk analysis functions for the value at risk of a portfolio [7].

4.3. Functions Used

4.3.1. cbind ( )

It is a function that puts R objects together either by combining them via columns or rows. It is considered as a generic function that goes along with other methods from other classes. It takes as arguments vectors and in case of data frames it has a modified version that is called rbind ( ) [8].

4.3.2. quantile ( )

It is a generic function that generates quantiles that match the provided probabilities. It takes as arguments vectors that are numeric and for which we want to calculate the sample percentiles [9].

4.3.3. tail( )

It is a generic function that returns the last part of an R object whether it is a function, a
matrix, a data frame or a table. It could as well be expanded to other classes [10].

4.3.4. \texttt{rnorm()} 

It is a function that randomly produces future portfolio values considering a normal distribution and taking as arguments a mean which is in our case the average of daily returns and a standard deviation of the portfolio returns as well and which were both calculated in the implementation of the Monet Carlo Simulation as will be shown in the appendix. It takes arguments such as a vector of probabilities, quantiles, of standard deviations, and of means and others [11].

5. \textbf{VaR Historical Simulation}

Historical simulation is a method used to estimate the value at risk. It consists of using old data to predict what will occur in the future.

Assume we want to calculate VaR for a portfolio using a time horizon of N= 1 day, a confidence level of 99% and 501 days of data (501 is a well-known choice for the number of days of the used data because it will result in 500 scenarios being generated) [4].

5.1. \textit{Steps of Historical Simulation}

The first step is to determine the market variables influencing the portfolio. They basically include equity prices, interest rates...The prices are all measured using the country’s primary currency. The collection of data is based on movements of the market variables in the recent 501 days. It generates 500 substitute scenarios for what may happen between today and tomorrow [4].

The first day for which we have data is defined as Day 0, the following as Day 1 and so on and so forth. For the first scenario, we assume that the percentage changes in the variables’ values are the same as they were between Day 0 and Day 1. The second alternative is when they were similar between Day 1 and Day 2... For each possible scenario, the portfolio value
change in dollar is computed. This way we could define a probability distribution for losses on a daily basis in the portfolio value [4].

The distribution’s 99th percentile is estimated as the highest 5th loss. Therefore, the VaR estimate is the loss located exactly at the 99th percentile point ➔ It means that we are 99% sure that we won’t incur a loss more than the VaR estimate condition that the changes in the variables in the recent 501 days accurately represent what will occur between today and tomorrow. To express the historical simulation approach using algebra, we define \( V_i \) as the value of the variable at Day I and assume that \( n \) Days have elapsed. The \( i \)th possible alternative in this approach supposes that the value of the variable tomorrow is [4]:

\[
Value \ under \ ith \ scenario = \theta_n \frac{\theta_i}{\theta_i - 1}
\]

5.2. Illustration of the Historical Simulation

In the historical simulation, I am using historical data of stock exchange indices to create different portfolios with assets (stocks) that belong to several indices and within that to different industries or companies. The aim of this section is to assess how the investments in different financial portfolios can lead to different results of the value at risk metrics. In other words, I am trying to see to which extent investing in a much diversified portfolio with assets from diversified sectors, i.e.: Pharmaceuticals, Textile & Luxury Goods, Financial Services, Consumer Goods, Automobiles Industry...

In the implementation part, I chose to work on two portfolios. The first one includes assets from the stock indices FTSE100, NASDAQ, and NYSE. The industries to which the stocks chosen belong are Health Care, Pharmaceuticals, and Oil & Gas Production respectively. The second portfolio consists of assets from indices S&P500, S&P100, and S&P400. It is more diversified in terms of industries as the assets chosen belong to the most broadly assets which include: Information Technology, Materials, Energy, Utilities, Health Care and Telecom.
5.3. Methodology to implement the Historical Simulation

The first thing was to start by loading the historical data for both portfolios from yahoo finance using the functions that exist in the Quantmod package. After loading the tickers and assigning weights, I used the “cbind” function to merge all the assets’ prices in one dataframe that I called “portfolioPrices”. Then, I computed the returns, i.e.: rates of change for each portfolio after deleting all the dates with no prices that would lead to NA values in the portfolioPrices dataframe. The last step was to compute the value at risk for each portfolio. The time horizon chosen was for the period from the 23rd of January to the 23rd of June for the year 2017 which is equivalent to N=152 days. The confidence level that I used in my calculations are X=99%, X=95% and X=68% for each portfolio VaR metrics.

5.4. Results and Interpretation

![Code Snippet](image)

Figure 2: VaR Results for the Historical Simulation of portfolios 1 & 2
In the screenshot above; we can see the results for the VaR metrics for each portfolio with the different confidence levels. From these findings, we can write the following conclusions:

- As for the first interval, we are 99% confident that we will not incur more than 1.39% loss in the 152 upcoming days for the first portfolio. Whereas for the second portfolio, we could have a 0.84% potential loss in the worst 1% of scenarios in N=152 days.

- For the second level in the first portfolio, we are 95% sure that we will not lose more than 1.34% in the coming 152 days whereas for the second portfolio we are sure that we might have a potential loss of 0.48% for the same time horizon.

- As for the last one, we are 68% confident that we will not incur more than 0.45% loss in the future 5 months for the first investment whereas for the second one we will not lose more than 0.13%.

- For the three value at risk metrics, we can see that the losses’ values are less in the second portfolio compared to the first one. We can, therefore, explain that by the fact that the second portfolio includes assets from more different sectors than the first one which means is more diversified. *We can say that diversified investments have less value at risks.*

### 6. Model-Building Approach

The model building approach is an important substitute to the historical simulation method. Before I explain the way this approach works, it is worth to mention the problem related to measuring volatility units [4].

**6.1. Daily Volatilities**

In pricing options, we measure time in terms of years most of the time and the asset’s volatility is also denoted as “volatility per year”. When we use the model-building approach to compute the value at risk, we usually measure time in days and thus the volatility is labeled as “volatility per day”. The question we ask here is: how does the volatility per year in the
pricing of options relate to the day volatility used in the VaR computations? Suppose we have $\sigma_{\text{year}}$ which is the volatility per year of some asset and $\sigma_{\text{day}}$ denotes the one-day volatility of the same asset. If we suppose we have 252 business days in one year, the following equation calculates the standard deviation of the return on the asset, which is continuously compounded, in one single year as $\sigma_{\text{year}}$ or as $\sigma_{\text{day}}\sqrt{252}$ [4].

$$\ln\frac{S_T}{S_0} \sim \Phi\left(\left(\mu - \frac{\sigma^2}{2}\right)T, \sigma^2T\right)$$

Where $S_0$ is the price of the stock at time $T=0$ and $S_T$ is the price at time $T$. From that we can write:

$$\sigma_{\text{year}} = \sigma_{\text{day}}\sqrt{252}$$

So it means that the volatility per day is around 6% of the yearly one (p.478, Hull). $\sigma_{\text{day}}$ nearly equals the standard deviation of the asset’s price percentage change in one single day. We assume precise equality when performing value at risk computations. Consequently, an asset price’s daily volatility is said to be equal to percentage change’s standard deviation in one day [4].

6.2. Case of a Single Asset

In the model building approach, the value at risk is calculated by considering a simple case in which the portfolio includes a particular position in a one single stock, i.e.: $10$ million in Microsoft company shares. Assuming that the time period is 10 days and the confidence level is $X=99\%$. To begin, we consider a one day time period [4].

Suppose that this stock’s volatility per day is 2%, which is around 32% in a year. Knowing that the size we choose for our position was $10$ million, the daily changes’ standard deviation should be about $200,000$. In this model, it is accepted to consider that a market variable’s expected change over some period of time is assumed to be zero. The change that
we expect in a market variable’s price during a short period of time is usually small as opposed to the change’s standard deviation [4].

For instance, assume that the expected return for Microsoft Company is 20% per year. Over one day, the expected return should be \( \frac{20\%}{252} \) that is 0.08% while the standard deviation is 2%.

For a period of 10 days, the expected return is 0.8% and the standard deviation is \( 2\sqrt{10} \) [4].

Up to now, we assumed that the change in Microsoft’s portfolio shares value over a period of one day has nearly an average of zero and a standard deviation of $200,000. Suppose the change has a normal distribution, and according to the Z tables \( N(-2.33) = 0.01 \), which means that there is a probability of 1% that this variable will be reduced by more than 2.33 standard deviations. Likewise, it also conveys that we are 99% confident that the same variable will not drop by more than 2.33 standard deviations [4].

Therefore, the value at risk for the portfolio that has $10 million in Microsoft stock with \( N=1 \) day and \( X=99\% \) is

\[
1 - \text{day VaR} = 2.33 \times 200,000 = \$466,000
\]

As mentioned previously, the N-day value at risk is computed as \( \sqrt{N} \) multiplied by itself for 1-day. Thus, the value at risk for Microsoft over a period of \( N=10 \) days assuming the same confidence level is [4].

\[
VaR(N = 10 \text{ days}) = 466,000 \times \sqrt{10} = \$1,473,621
\]

As a second example, we suppose we have a portfolio of $5 million position in the company of AT&T, and assume that the volatility per day is 1% meaning 16% in a year. Similarly to our previous calculations for Microsoft, we compute the one day standard deviation of the portfolio value change as [4]:

\[
SD = 5,000,000 \times 0.01 = \$50,000
\]

Again, if we suppose normal distribution of the change, the value at risk for one day with a level of \( X = 99 \% \) is:


\[ 1 - Day VaR = 50,000 \times 2.33 = \$116,500 \]

And for N=10 days:

\[ 10 - Day VaR = 116,500 \times \$368,405 \]

**6.3. Two-Asset Case**

Assume we have a portfolio that includes $5 million and $10 million shares from AT&T and Microsoft respectively; consider that both shares returns have a correlation of 0.3 in a bivariate normal distribution. According to statistics, if we have two variables X and Y have standard deviations \( \sigma_x \) and \( \sigma_y \) with a correlation coefficient equal to \( \rho \), the total standard deviation of X+Y is [4]:

\[
\sigma(x + y) = \sqrt{\sigma^2 x + \sigma^2 y + 2\rho \sigma x \sigma y}
\]

So as to apply the above formula, we take X as the change in Microsoft position value over a period of one day and Y equals that of AT&T over the same period, so we get:

\[ \sigma(x) = 200,000 \quad and \quad \sigma(y) = 50,000 \]

Therefore, the standard deviation of the two stocks portfolio value change is computed as [4]:

\[ SD = \sqrt{200,000^2 + 50,000^2 + 2 \times 0.3 \times 200,000 \times 50,000} = 220,227 \]

The change in the mean is considered to be null and the change in the portfolio value is assumed to have a normal distribution. Thus, the value at risk for one day with X = 99% is:

\[ VaR = 220,227 \times 2.33 = \$513,129 \]

And for a time horizon of N=10 days [4]:

\[ VaR = 513,129 \sqrt{10} = \$1,622,657 \]

**6.4. The Benefits of Diversification**

Based on the previous examples, we have [4]:

a) The value at risk for Microsoft shares portfolio (N=10 days and X=99%) is $1,473,621.

b) The value at risk for AT&T shares portfolio (N=10 days and X=99%) is $368,405.
c) The value at risk for the diversified portfolio (N=10 days and X=99%) is $1,622,657.

In order to calculate the amount of diversification benefits, we do:

\[(1,473,621 + 368,405) - 1,622,657 = $219,369\]

If the stocks of AT&T and Microsoft had a perfect correlation, the value at risk of the diversified portfolio would be exactly equal to the sum of the values at risk of each of the two stocks (p.480, Hull). When we have a little less than a perfect correlation of two variables, then it induces some risk to be “diversified away” [4].

7. Linear Vs Quadratic Model

7.1. Linear Model

The previous examples simply portray how we use the linear model approach for computing the Value at Risk. Consider a portfolio P that has n assets with a particular amount \(\alpha_i\) that is invested in an asset i such that \(1 \leq i \leq n\); we suppose \(\Delta x_i\) is the one day return on asset i [4]. The investment’s value change in dollars in on day is denoted by \(\alpha_i \times \Delta x_i\) and generally speaking we write:

\[\Delta P = \sum_{i=1}^{n} \alpha_i \times \Delta x_i\]

Where \(\Delta P\) is the change in the total portfolio value for one day. In we apply this to the examples provided in the previous section, assuming the $10 million invested in Microsoft stock and the $5 million in the AT&T one, we get \(\alpha_1 = 10\) and \(\alpha_2 = 5\) (in millions of dollars) [4]:

\[\Delta P = 10\Delta x_1 + 5\Delta x_2\]

Suppose the \(\Delta x_i\) is multivariate and normally distributed, then the change \(\Delta P\) is also has a normal distribution. In order to compute the value at risk, we only have to compute the standard deviation and the average of \(\Delta P\). As mentioned previously, we suppose that each \(\Delta x_i\) has a null expected value which means that the average of \(\Delta P\) is null as well [4].
To find the standard deviation of the whole portfolio value change $\Delta P$, we suppose we have $\sigma_i$ as being the volatility per day of asset i and we assume $\rho_{ij}$ as the correlation coefficient between the returns on both assets i and j. That is to say, $\sigma_i$ is the standard deviation of asset $\Delta x_i$ and $\rho_{ij}$ is the correlation coefficient between assets $\Delta x_j$ and $\Delta x_i$; the variance of the change $\Delta P$ is $\sigma^2_p$ and is computed by the formula below [4]:

$$\sigma^2_p = \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij}\alpha_i\alpha_j\sigma_i\sigma_j$$  \hspace{1cm} (21.2)

The above formula could also be written as follows [4]:

$$\sigma^2_p = \sum_{i=1}^{n} \alpha^2_i\sigma^2_i + 2\sum_{i=1}^{n} \sum_{j<i}^{n} \rho_{ij}\alpha_i\alpha_j\sigma_i\sigma_j$$

Generally speaking, the change’s standard deviation for a period of N days is denoted by $\sqrt{N} \times \sigma_p$, therefore, the value at risk for N days with a confidence level of 99% is $2.33 \times \sqrt{N} \times \sigma_p$. For one day, the return of the portfolio is $\Delta P/P$. From the previous equations, the variance of it should be [4]:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij}\omega_i\omega_j\sigma_i\sigma_j$$

The $\omega_i = \alpha_i/P$ is assumed to be the weight of the investment i in the portfolio. The equation above is the one that is most of the time used by financial managers. In the examples considered previously, we have $\sigma_1 = 0.02$ and $\sigma_2 = 0.01$ and $\rho_{12} = 0.3$ and as mentioned before $\alpha_1 = 10$ and $\alpha_2 = 5$, so the variance is [4]:

$$\sigma^2_p = 10^2 \times 0.02^2 + 5^2 \times 0.01^2 + 2 \times 10 \times 5 \times 0.3 \times 0.02 \times 0.01 = 0.0485$$

And the standard deviation of the portfolio value change for one day is therefore: $\sigma_p = 0.220$.

The Value at risk for N= 10 days and X= 99 % is $\text{VaR} = 2.33 \times 0.220 \times \sqrt{10} = \$ 1,623 million$; it matches the findings in the previous section [4]:

### 7.2. Correlation and Covariance Matrices
We call a matrix as a correlation matrix when the entry in the row i and the column j corresponds to the correlation \( \rho_{ij} \) between both variables j and i. Because a variable all the time has a perfect correlation with itself, the elements in the diagonal of the correlation matrix always take values of 1. Besides, this matrix is always symmetric since \( \rho_{ij} = \rho_{ji} \). If we have the data from the correlation matrix along with the variables’ standard deviations per day, we could compute the variance of the portfolio using the formula in (21.2). Most of the time, financial managers use variances and covariances instead of volatilities and correlations. The variance per day is \( \text{var}_i \) of a variable i is its volatility squared and we write [4]:

\[
\text{var}_i = \sigma^2_i
\]

The covariance \( \text{cov}_{ij} \) between two variables j and i is the product of the volatility per day of variable j, the volatility per day of variable i, and the correlation coefficient between both i and j, the formula is [4]:

\[
\text{cov}_{ij} = \sigma_i \sigma_j \rho_{ij}
\]

From that we could derive the equation for the portfolio value change variance as follows [4]:

\[
\sigma^2_p = \sum_{i=1}^{n} \sum_{j=1}^{n} \text{cov}_{ij} \alpha_i \alpha_j \quad (21.3)
\]

\[
\begin{bmatrix}
1 & \rho_{12} & \rho_{13} & \cdots & \rho_{1n} \\
\rho_{21} & 1 & \rho_{23} & \cdots & \rho_{2n} \\
\rho_{31} & \rho_{32} & 1 & \cdots & \rho_{3n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\rho_{n1} & \rho_{n2} & \rho_{n3} & \cdots & 1
\end{bmatrix}
\]

Figure 3: A correlation matrix where \( \rho_{ij} \) is the correlation coefficient between both i and j.
Figure 4: A variance-covariance matrix

In the covariance matrix, the matrix entry in row i and column j is the covariance between both i and j. The diagonal entries represent the variances which are in fact the covariances between the variables and themselves. That is why this matrix could also be called variance-covariance matrix. Just like the correlation one, the covariance matrix is symmetric. If we want to rewrite the previous equation in (21.3) using matrix notation, the formula for the portfolio’s standard deviation becomes [4]:

$$\sigma^2_p = \alpha C \alpha$$

α is the vector of columns which has αi as the ith element, C is the covariance matrix, and α is α’s transpose. Covariances and variances are most of the time computed from assets’ historical data [4]:

**7.3. Applications of the Linear Model**

The linear model approach could be simply applied to a portfolio where there are no derivatives and that has many investments in different assets such as bonds, commodities, stocks, and foreign exchange. In this situation, the portfolio value has a linear change that depends as well on those assets’ prices’ changes (p.484, Hull). For reasons of computing the value at risk, all the prices of assets should be in the same currency, domestic currency. For instance, some large bank in the United States would consider market variables that would consist of, for example, the value of the Nikkei index in US dollars, the price of a 5-years zero coupon sterling bond also in dollars and so on and so forth [4].

**7.4. Quadratic Model**
The linear model does not always give precise results, for example, if a portfolio consists of options along with other assets, then the model would only give an approximate value to the result. The reason behind that is that it doesn’t take into consideration the gamma value of the portfolio. Delta determines the change rate of the value of the portfolio with respect to an implicit market variable whereas gamma defines the rate of change of delta itself with respect to the variable. This latter measures how the relationship between the value of the portfolio and the implicit market variable is curved [4].

**Figure 21.4** Probability distribution for value of portfolio: (a) positive gamma; (b) negative gamma.

![Figure 21.4](image)

**Figure 5: Probability distribution function of the portfolio value**

The figure above illustrates the effect of a gamma value that is different than zero on the probability distribution of the portfolio value. When it is positive, the distribution is likely to be positively altered whereas when gamma takes a value that is less than zero, then the probability distribution is negatively altered [4].

The following figures show the reasons of the results obtained in Figure 21.4.
Figure 6: Probability distribution for a value of a long call on an asset

Figure 7: Probability distribution for a value of a short call on an asset
Figure 21.5 illustrates the relationship between the price of an underlying asset and the value of an option of long call. A long call represents a position of an option that has a positive gamma value. The figure illustrates that when the distribution of the probability for the underlying asset’s price is normal at the end of the first day, the probability distribution of the price of the option is positively altered (the normal distribution could be used to approximate the lognormal distribution for purposes of calculating the VaR) [4].

Figure 21.6 illustrates how a short call position value relates to the price of the asset. Unlike the long call position, the short call position has a negative gamma value. In this situation, we notice that the normal distribution of the asset’s price at the end of the first day gets matched to a probability distribution that is negatively distorted for the option position value [4].

We already know that the value at risk of a particular portfolio depends to a great extent on the left tail of the portfolio value probability distribution. For instance, if the level of confidence is 99%, the value at risk lies in the value in the distribution’s left tail below which there is only 1% left of value distribution. As mentioned in the Figures 21.4a and 21.5, when the portfolio has a positive gamma value, it is more likely to have a less substantial left tail compared to the normal distribution. When the probability distribution of ΔP in a normal one, the value at risk of the portfolio is likely to be very large. Likewise, as it is explained in Figures 21.4b and 21.6, the portfolio with a negative gamma seems to have a left tail that is much weighty than the normal distribution. And in this second situation, if the ΔP’s distribution is normal, the value of the value at risk is very low [4].

In order to get a more precise estimation of the value at risk than the one provided in the previous section by the linear model, financial managers could use delta and gamma methods to relate ΔP to Δxi. Assume we have a portfolio that consists of a single asset with price S. Consider δ and γ respectively as delta and gamma of the chosen portfolio. Generally speaking, if we have a portfolio with n market variables, with each portfolio instrument depends only on one of the market variables, we write [4]:

...
\[ \Delta P = \sum_{i=1}^{n} S_i \delta_i \Delta x_i + \sum_{i=1}^{n} \frac{1}{2} S_i^2 \gamma_i (\Delta x_i)^2 \]

\( S_i \) is the value of the market variable \( i \), and \( \gamma_i \) and \( \delta_i \) are the gamma and delta of the portfolio with respect to that same market variable. In case the instruments in the portfolio depend on more than one variable of the market, the previous formula becomes as follows [4]:

\[ \Delta P = \sum_{i=1}^{n} S_i \delta_i \Delta x_i + \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{2} (S_i S_j) \gamma_{ij} (\Delta x_i \Delta x_j) \]  \hspace{1cm} (21.7)

Where \( \gamma_{ij} \) is a “cross gamma” denoted by the following equation:

\[ \gamma_{ij} = \frac{\partial^2 P}{\partial S_i \partial S_j} \]

The equation 21.7 is hard to apply as a formula, however, it could be used to compute the moments of \( \Delta P \) [4].

8. Monte Carlo Simulation

8.1. Definition

Monte Carlo Simulation could be considered as a substitute to the procedure described in the previous two sections, meaning that it is possible to implement the Model-Building approach by using the Monte Carlo simulation to produce the portfolio change’s (\( \Delta P \)) probability distribution. Assume that we want to compute the daily value at risk for a specific portfolio. The steps are the following [4]:

1) Find today’s portfolio value following the typical way by using the current market variables’ values.

2) Take one sample from the \( \Delta x_i \)’s probability distribution that is, of course, a multivariate and normal one.

3) Define each market variable value, by the end of the day, by using the sampled values of the \( \Delta x_i \).

4) Reevaluate the value of the whole portfolio for the end of day as usual.
5) Determine the sample $\Delta P$ as the difference between the values computed in parts 1 and 4.

6) Keep repeating the steps from 2 to 5 in order to generate the probability distribution for the whole portfolio change, i.e.: $\Delta P$.

The value at risk is afterwards computed as the most correct percentile of $\Delta P$’s probability distribution. Assume, for instance, that 5000 distinct $\Delta P$ sample values were calculated in the same way described in the procedure. Assuming X=95% and N=1 day, the value at risk is the result of $\Delta P$ for the 50th worst output; for the same time horizon and assuming X=95%, the value at risk is equal to $\Delta P$ value for the 250th worst output and so on and so forth. As mentioned before, the value at risk for N number of days is said to be the value at risk for one day multiplied by $\sqrt{Number\ Of\ Days}$ [4].

There is one disadvantage of Monte Carlo simulation which is that it is likely to be very slow because the whole portfolio of a company should be evaluated and reevaluated many times, keeping in mind that it may include a huge number of several instruments [4].

In order to make the process faster is by assuming that the equation (21.7) describes the relationship between $\Delta x_i$ and $\Delta P$. It will allow us to go directly from step two to 5 in the Monte Carlo process steps and thus we will not need to implement an exhaustive evaluation of the company’s portfolio. This is what we refer to as “Partial Simulation Approach”. Similarly, there is an approach that could also be used when doing the historical simulation [4].

**8.2. Implementation of Different Portfolios**
In the implementation part of the Monte Carlo Simulation, I chose to use two portfolios for which I will compute the value at risk using this method. I have chosen stocks from the S&P 100 assets and for the second one, the assets belong to the Dow 30 index. The way I created the portfolios applies to both. I started with getting the prices for the stocks using a loop, then deleting all dates that have no prices. This is to avoid having missing data in our time series that may cause some required functions not to work. Then, I had to get the portfolio’s total value by multiplying the share amounts by the prices to get the amount invested in each asset. Then, to get the portfolio value we sum the prices per row in a column using function the “cbind ()” then storing the result in a dataframe called “portPrices”.

We need to generate future portfolio values in the Monte Carlo simulation to estimate Value at Risk. In order to do that, we need a portfolio value to start with (i.e. the last value of the portfolio we chose to use) and an average of daily returns and a standard deviation to go off of. Finally, we calculate the returns.

The next step was to run the Monte Carlo Simulation for the stock prices. For that, we need to get the standard deviation of the portfolio returns using the function “apply ()” that takes the “Total” from the portfolio returns as a first parameter, and as the margin parameter takes the value “2” which means that we are calculating the standard deviation based on the columns rather than the rows, and the last parameter which is the function we use “sd” which means the function of the standard deviation. We, then get the mean which is the average of the daily returns. Then, we define the number of simulations and which I chose to be 100 and then the number of days’ worth of future prices in simulations which I chose to be 50. The way I chose to perform the Monte Carlo simulation is by simply using the “rnorm” function which is a function that randomly generates future prices from a normal distribution. We use a loop to create the simulations and inside of it we append the original price which is the last price of the portfolio which we extracted using the “tail ()” function to the future prices’ vector. In each simulation inside that loop, we extract the last price and append it to the new list. After I
am done with the prices’ simulations, I created a dataframe that to which I appended each one of them. The last thing I did was to rename the columns in the simulations dataframe where each one has its own number following the order in which it was generated.

The next step was to calculate the value at risk metrics. In order to do that, I had to extract the final prices again using the same “tail ()” function and storing them in a vector of prices which I called “future_prices” then sorting them in ascending order. Vectors of returns or prices should normally be sorted before determining the price at the given confidence level.

This is true for historical simulations as well. Next, I generated the stock prices at different percentiles using the “quantile ()” function and stored them in variables p1, p2, p3 at confidence levels of 95%, 99%, 90% respectively. Then, to get the value at risk at each level, we just subtract the last price that we computed earlier from the previous stock prices p1, p2, and p3. The results generated are in Dollar amounts and could also be generated in percentages.

**S&P Portfolio**

For this portfolio we have assets from different indices from S&P. We got the following results shown in the screenshot below.

![Figure 8: VaR calculation implementation in Monte Carlo Simulation for S&P portfolio](image-url)
The results shown are in Dollars amount. For the first confidence level, we are 95% sure that we will not lose more than $7587.274 in the next 50 days. For the second one, we know that the potential amount that we could incur as a loss is $10687.68 in the worst 1% of days. And for the last level, we are 90% that we will not lose more than $5417.883 in the next 50 days. Results for the same portfolio were also generated in percentages. That could be simply done by dividing the Monte Carlo value at risk metrics by the portfolio value. The results are shown in the screenshot below.

We can see that for the first interval, we are 95% sure that we will not incur more than 5.31% loss in the upcoming 50 days. For the second one, we are 99% that we will not lose more than 7.48% for the same time horizon. And for the confidence level, we are 90% that we
will not lose more than 3.79 % for the same N = 50 days. From the results explained above, we could deduct the following about the losses in dollars and percentages:

1- A loss of $7587.274 corresponds to a value at risk of percentage 5.31 % for N = 50 days and an interval of 95 %.

2- Similarly, a loss of $10687.68 corresponds to a value at risk of percentage 7.48 % for N = 50 days and X = 99%.

3- Finally, a loss of $5417.883 corresponds to a value at risk of percentage 3.79 % for N = 50 days and X = 90 %.

**Dow 30 Portfolio**

For the Dow 30 portfolio, we have the following results shown in the screenshot below.

![Figure 11: VaR ( Dollars ) calculation implementation for Dow 30 portfolio](image1)

For the first confidence level, we are 95 % sure that we will not lose more than $1527.673 in the next 50 days. For the second one, we know that the potential amount that we could incur...
as a loss is $2809.909 in the worst 1 % of days. And for the last level, we are 90 % that we will not lose more than $1089.485 in the next 50 days.

Figure 13: VaR (percentages %) Calculation Implementation for Dow 30 portfolio

Figure 14: VaR Results for the Dow 30 portfolio in percentages

From the VaR results generated percentages, we see that for the first interval, we are 95 % sure that we will not incur more than 4.43 % loss in the upcoming 50 days. For the second one, we are 99 % that we will not lose more than 8.16 % for the same time horizon. And for the confidence level, we are 90 % that we will not lose more than 3.16 % for the same N = 50 days. From the results explained above, we could deduct the following about the losses in dollars and percentages:

1- A loss of $1527.673 corresponds to a value at risk of percentage 4.43 % for N = 50 days and an interval of 95 %.

2- Similarly, a loss of $2809.909 corresponds to a value at risk of percentage 8.16 % for N = 50 days and X = 99%.
3- Finally, a loss of $1089.485 corresponds to a value at risk of percentage 3.16% for N = 50 days and X = 90%.

**Portfolio 3**

This portfolio includes stocks from NYSE and FTSE and NASDAQ and is the same one used previously in the historical simulation section. However, the industries are not very diversified unlike the S&P portfolio. The results are shown below:

Figure 15: VaR Calculations Implementation for the third portfolio in MC Simulation

```r
> VAR1
[1] -542.876
> VAR2
[1] -6875.941
> VAR3
[1] -4146.11
> #Calculate Monte Carlo VaR for Portfolio 3 in Percentages
> VAR_per1 <- p1[[1]] / lastPrice
> VAR_per1 <- VAR_per1/lastPrice
> > VAR_per2 <- p2[[1]] / lastPrice
> VAR_per2 <- VAR_per2/lastPrice
> > VAR_per3 <- p3[[1]] / lastPrice
> VAR_per3 <- VAR_per3/lastPrice
> > VAR_per1
[1] -0.0728688
> VAR_per2
[1] -0.09039378
> VAR_per3
[1] -0.05450637
```
Figure 16: VaR Results for the third portfolio in Dollar amounts ($) and percentages (%)

From the results above, we could deduce the following about the losses in dollars and percentages for this last portfolio:

We are 95% confident that we will not incur a loss of more than $5,542.876 in the future 50 days which corresponds to a value at risk percentage of 7.28%.

For the second interval, we are 99% sure that the portfolio potential loss in the next 50 days will not exceed $6,875.941 that corresponds to a value at risk percentage of 9.03%.

For the last confidence level, we are 90% sure that we will not lose more than $4,146.11 in the upcoming 50 days which corresponds to a value at risk percentage of 5.45%.

8.3. Results and Findings

In the methodology of this project, I used two methods for measuring risk in financial portfolios which are Historical Simulation and Monte Carlo Simulation. For both methods I used different portfolios which consisted each of stocks as assets. The results that both methods generated showed that the diversified portfolios had smaller values for the value at risk. The first portfolio consisted of assets from FTSE, NASDAQ, & NYSE indices and belonged to sectors of Health Care, Oil & Gas Production and Pharmaceuticals. Whereas the second portfolio included assets from S&P 100, S&P 400, and S&P 500 which were from different sectors of Health Care, Internet Technologies, Utilities, Materials, Telecom, Financials and Consumer Staples. Therefore, the S&P portfolio is much more diversified than the first one. The results of the Monte Carlo Simulation confirm what we found earlier in the results of the Historical Simulation which are that value at risk values are small in portfolios that are more diversified and the screenshots below prove it.

For the first portfolio, we got the following findings for the value at risk:
For the second one, we have the following values:

As we can see, the second portfolio of the S&P has smaller values for each confidence level compared to the first portfolio.

- For \( X = 95\% \), we have a value of \( \text{VaR} = 7.28\% \) for the first portfolio whereas the second one has a \( \text{VaR} = 5.31\% \).
- For \( X = 99\% \), we have a value of \( \text{VaR} = 9.03\% \) for portfolio 1 whereas we have a \( \text{VaR} = 7.48\% \) for the second diversified portfolio.
- For \( X = 90\% \), the first portfolio has a \( \text{VaR} = 5.45\% \) whereas the second one has a \( \text{VaR} = 3.79\% \).

We can conclude that it is less risky to invest in diversified portfolios that include assets from different industries. Therefore, diversification is a good way to reduce risk in our portfolio.

**8.4. Comparison of Approaches**

So far in this report, two methods for calculating the value at risk were explained. They are the Historical Simulation and the Model-Building approaches. The benefit of the Model-building method is that the results of it can be quickly generated and that it could be
conveniently used in combination with volatility updating practices. However, the main
drawback of this approach is that it considers a multivariate normal probability distribution of
market variables used [4].

Practically speaking, the market variables undergo changes every day; that is why they often
have probability distributions with tails that largely differ from the normal one. Moreover, the
model-building approach is very likely to generate poor findings for financial portfolios with
low-delta values [4].

The historical simulation method has the convenience that it defines market variables joint
probability distribution. Besides, when using the Historical Simulation, financial managers no
longer need to perform the cash-flow mapping. The major inconvenient of this method is that
it is slow when performing computations and does not give much freedom in combining it
with volatility updating practices [4].
9. Conclusion

This project would provide financial analysts and portfolio managers with a program that would help them a lot in assessing risk degrees in their portfolios. It combines knowledge and techniques from fields of mathematics, finance, and statistical programming that required a lot of research to be gathered. The program implements calculations of value at risk using two main different methods which are the Historical Simulation and the Monet Carlo Simulation. Portfolio managers could use it to assess the risk in their portfolios that consist mainly of stocks as these were the main assets chosen to work with in this project. An important finding they would come up to while using this program is that they have to diversify their investments/portfolios in order to minimize the risk which was confirmed by the results of both measures’ implementations. Explanations are given in the demo part which will allow any person who reads this report, even with no prior knowledge in the field, to understand what it does. For future work, we could consider implementing other methods to calculate the Value at Risk and drawing a comparison between more methods to evaluate the effectiveness of each. Also, one could try to include other sorts of assets such as bonds and options and try to assess the risk in a portfolio with more varying assets.
10. References


11. Appendix/Demo

In this section, I am including the code snippets that show the execution of the R program step by step from launching the R-Studio environment to implementing the VaR calculations then displaying the final results for each method.

First, here is what the R Studio IDE looks like:

![R Studio IDE](image)

**Figure 19: R Studio IDE**

Then, I switch to the Capstone Project and get the following:
Then, we load the first R script that explains the Historical Simulation where we start with loading the required packages of Quantmod and PerformanceAnalytics then proceed with the implementation for portfolios 1 & 2 as explained previously in the methodology of chapter 5 and shown in the screenshots below.
The prices for this portfolio 1 are as follows:

![Figure 22: Stocks’ Prices for Portfolio 1](image)

And the following results:
Similarly, we have the implementation for portfolio 2 of S&P.
Figure 24: Historical Simulation Script for Portfolio 2

And we got the following S&P prices and the following VaR metrics:

Figure 25: Stocks’ Prices for Portfolio 2 with VaR Metrics Results

Moving to the Monte Carlo Simulation, we have for the first portfolio of S&P:
Similarly, for the second portfolio of Dow 30:

![Monte Carlo Simulation for Portfolio 2](image)

Figure 27: Monte Carlo Simulation for Portfolio 2

Similarly, for the third portfolio:
Afterwards, we run the following piece of code for each of the portfolios to get the value at risk metrics both in Dollar amounts, and then in percentages. The results for each portfolio were provided in the second and third section of chapter 8 with detailed explanations.

Figure 28: Monte Carlo Simulation for Portfolio 3

Figure 29: Monte Carlo Simulation Implementation
Figure 30: Monte Carlo Simulation Implementation Follow-up

```r
# Loop to create simulations
47
for (i in 1:nseed) {
48
  # Create Vector of Prices
49  y <- c()
50
  # Append original price
51  y <- c(y, lastprice)
52
  # Generate future prices
53  for (j in 1:predicted_days)
54
    # Create New Price
55    rnorm is a function to randomly generate future prices assuming a normal distribution with a mean of
56    new_price = rnorm(1, mean=avg_ret, sd=vol)
57
    # Append price to vector
58    y <- c(y, new_price)
59
  # Create dataframe
60  if (i == 1) {
61    df = data.frame()
62  } else {
63    # Append simulation to dataframe
64    df <- cbind(df, y)
65  }
66
254 (Top Left)
```

Figure 31: Monte Carlo Simulation Implementation Follow-up

```r
# Rename columns
for (i in 1:ncol(df)) {
  col_name <- paste("sim", tostring(i))
  names(df)[i] <- col_name
}

# Calculate VAR
# Extract Final Prices
future_prices <- tail(df, 1)
# Sort in Ascending Order
future_prices <- sort(future_prices)
# Get Stock Prices at different percentiles
quantile fnc produces sample quantiles corresponding to the given probabilities.
p1 <- quantile(future_prices, probs = .05)  # 95% confidence level
p2 <- quantile(future_prices, probs = .05)  # 90% confidence level
p3 <- quantile(future_prices, probs = .10)  # 80% confidence level
# Calculate monte carlo var for portfolio in dollar amounts
VARI1 <- p1[[1]] - lastprice
VARI2 <- p2[[1]] - lastprice
VARI3 <- p3[[1]] - lastprice
VAR1
VAR2
VAR3
```
Figure 32: Monte Carlo Simulation Implementation Follow-up