

# Motion from Ordered Sets of Curvature Points

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## Abstract

*This paper presents a technique for determining the parameters of rigid or affine 2D motion of boundaries, by sequentially building point pairs, and checking the consistency of locally determined motions. The parameters of the motion are estimated from the positions of points of significant curvature, rather than from curvature values as is done by other techniques. This novel technique is robust and copes particularly well with occlusion problems at curve ends. In particular, experimental tests show that our technique is more accurate than the Hough Transform in measuring motion parameters.*

## 1 Introduction

An important task in computer vision and image processing is to determine the transformation which maps a 2-D curve into another. Such transformations are of crucial importance to 2-D recognition [4, 13, 2], motion and tracking. To this end, points of significant curvature [15, 5] have received special attention in the computation of the parameters of the transformation due to their local nature.

Many methods have been proposed to evaluate the parameters of the transformation of a 2-D curve into another. Such techniques can be classified into local and global: the latter are based on matching a global vector of features, and thus suffer from occlusion [11]; the former use local features (such as curvature), which depend only upon portions of objects [1, 6].

This work aims at computing motion parameters of object boundaries from points of significant curvature. Boundaries are first extracted from sequences of images [10], and their correspondence is established [9]. Points of significant curvature are computed from curvature profiles of each pair of corresponding boundaries. This suggests a natural coarse-to-fine approach for estimating motion in a wide variety of circumstances.

## 2 Related work

Cohen *et al.*, proposed a method for determining transformations between two curves. This method is based on the minimisation of energy which tends to preserve the matching of points of significant curvature, while ensuring a smooth field of displacement vectors everywhere [3]. However, this technique is suited for deformable curves, and is computationally very expensive since it requires the solution of a second order partial differential equation. Furthermore, the method also requires a first guess to start the iterative solution of the differential equation, and does not explicitly solve for the parameters of the transformation.

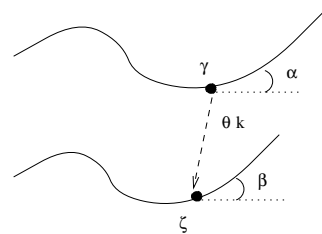


Figure 1. Curve  $C$  transforms to  $C'$ .  $\alpha$ ,  $\beta$  and  $\gamma$ ,  $\zeta$  are the slope angles, and curvatures of corresponding points.

Ma and Chen's technique is based on a Generalised Hough transform [7]: for a curve  $C$  undergoing a transformation to become  $C'$  (see **Figure 1**), the slope angles  $\alpha$  and  $\beta$ , and the curvatures  $\gamma$  and  $\zeta$  on corresponding points are related by:

$$\text{tg}(\alpha - \beta) = \text{tg}(\theta) \quad \text{and} \quad \zeta = \frac{1}{k}\gamma$$

where  $\theta$  is the rotation angle, and  $k$  is the expansion factor. Ma and Chen's algorithm consists of computing  $(k, \theta)$  using the above equations for every pair of points of the curves  $C$  and  $C'$ , and incrementing the corresponding location in an accumulator array  $B(k, \theta)$ . Local maxima in  $B(k, \theta)$  correspond to the parameters of the transformation.

There are a number of intrinsic problems with this approach. Namely, its time complexity is very high, and

corners, where curvatures are undefined, yield wrong parameters. The latter problem, actually, reflects the locality of the technique. In other words, through a small aperture placed over the corner, it is impossible to say if an expansion is occurring or not. Of course, if there is another point of high curvature within the aperture, then the combination of local measurements at these points can yield the expansion rate. This is, the aperture problem [14]. Furthermore, the sensitivity of curvature to perturbations and noise, causes the obtained values for  $k$  (ratio of curvatures) to lack precision;

### 3 Motion Model

For a given pair of boundaries  $C$  and its transform  $C'$ , let  $(m, n) \in C \times C'$  be a pair of corresponding points on the continuous curves  $C$  and  $C'$ . Asserting that  $m = (x, y)^T$  corresponds to  $n = (u, v)^T$  gives:

$$\begin{aligned} x &= a^{(0)} + a^{(1)}u + a^{(2)}v \\ y &= a^{(3)} + a^{(4)}u + a^{(5)}v \end{aligned} \quad (1)$$

where  $a^{(i)}$ ,  $i \in \{0..5\}$ , are the six unknown parameters of the general affine motion model. Such transformation includes rotation, translation, scaling and skewing.

In this work, we limit ourselves to the following restricted model:

$$\begin{pmatrix} x \\ y \end{pmatrix} = kR \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \quad (2)$$

where  $k$  is the expansion factor,  $X_0 = (x_0, y_0)^T$  is the translation vector and  $R$  is the rotation matrix:

$$R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

As will be shown in the results section, the model given in (2) captures well the motions involved in the contexts we are interested in (*i.e.* conveyor belt and independently moving objects with small rigid or affine changes between successive frames).

### 4 Establishing Motion Parameters

In perceiving the motion of **Figure 2**, it appears that the human visual system uses the distances between corners of each curve to work out the expansion, and uses the orientation of the lines joining these corners to derive the rotation<sup>1</sup>. Looking at the locations of all points

<sup>1</sup>We have no proof for these claims. This matter may be a fruitful topic of research in psychology, in which measurements of perception of expansion and rotation using curvature on the one hand, and positions of high curvature points, on the other hand, are compared.

of significant curvature within the same boundary is a global approach which overcomes the aperture problem.

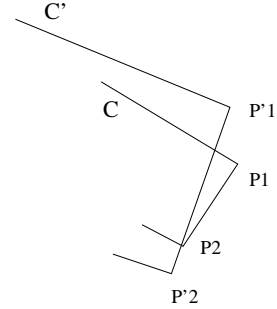


Figure 2. The curve  $C'$  was obtained by rotating and expanding the curve  $C$ .

Based on this remark, the parameters of the transformation can be defined as follows:

$$k = \frac{\|P'1\vec{P}'2\|}{\|P1\vec{P}2\|} \quad (3)$$

$$\theta = \text{acos} \left( \frac{P'1\vec{P}'2 \cdot P1\vec{P}2}{\|P'1\vec{P}'2\| \cdot \|P1\vec{P}2\|} \right) \quad (4)$$

$$X_0 = P'1 - kR.P1 \quad (5)$$

where  $k$  is the expansion factor,  $\theta$  the angle of rotation, and  $X_0$  the translation vector.

What seems important then is not so much the value of the curvature at the corners, but the locations of these corners – points of maximum curvature, in general. It follows that, if coherent correspondence between high curvature points of  $C$  and  $C'$  is established, the parameters  $k, \theta$ , and  $X_0$  can be computed using the above equations.

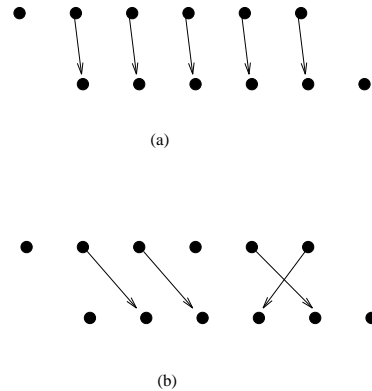


Figure 3. Bullets designate significant curvature points. Arrows indicate pairings. (a) allowed configuration. (b) disallowed configuration because of the crossing.

Based on the above findings, the computation of motion parameters is posed as a matching problem, that is, establishing correspondence between points of significant curvature. Then, motion parameter values are computed from the resulting coherent pairings.

## 5 Reformulating the Problem

The correspondence must be a one-to-one, and without crossings (see **Figure 3**). The first constraint (*i.e.*, one-to-one) stems from the physical nature of points of high curvature, *i.e.*, a point of high curvature in the scene always projects into a unique point in the image plane. The second constraint (*i.e.*, prohibition of crossings) is dictated by the natural ordering on boundary points. The ordering not only allows straightforward processing and measurements, but must also be satisfied by all plausible solutions.

Additionally, if such matching exists for a solid boundary undergoing a transformation, consistency must be conserved between pairs of corresponding high curvature points. Let  $P_1, P_2, \dots, P_n$  be the ordered points on  $C$ , and let  $P'_1, P'_2, \dots, P'_n$  be their corresponding high curvature points on  $C'$ . Let

$$k_{i,j} = \frac{\|P'_i P'_j\|}{\|P_i P_j\|} \quad \forall i, j \in \{1..n\}, \quad i \neq j \quad (6)$$

$k_{i,j}$  represents the expansion that the curve  $C$  undergoes to become  $C'$ , estimated here from  $P_i, P'_i, P_j$  and  $P'_j$ . Because of the rigidity assumption, the expansion factor is the same for all pairs of corresponding points (see **Figure 4**).

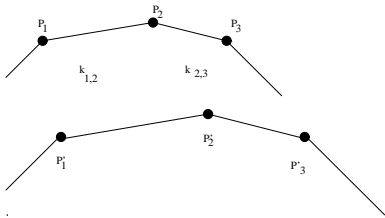


Figure 4. *Expansion factors  $k_{1,2}$  and  $k_{2,3}$  are consistent i.e.,  $k_{1,2} = k_{2,3}$ . Similarly the angles of rotation are equal i.e.,  $\theta_{1,2} = \theta_{2,3}$ .*

Thus, the consistency constraint may be written as follows:

$$\forall i, j \in \{1..n\}, \quad i \neq j, \quad \forall h, l \in \{1..n\}, \quad h \neq l, \quad k_{i,j} = k_{h,l} \quad (7)$$

In practice, such a constraint is very strong, and has to be relaxed to allow for small differences between the

different  $k_{i,j}$ . Moreover, it is not necessary to consider all pairs of points  $P_i, P'_i, P_j, P'_j, P_h, P'_h, P_l$  and  $P'_l$ . It can be shown that the subset of successive points  $P_i, P'_i, P_{i+1}, P'_{i+1}, P_{i+2}$  and  $P'_{i+2}$  gives a sufficient constraint. In the light of the above remark, the consistency constraint is then expressed as follows:

$$\forall i \in \{1..n-2\}, \quad |k_{i,i+1} - k_{i+1,i+2}| \leq \epsilon_1 \quad (8)$$

where  $\epsilon_1$  represents the maximum difference allowed. Typically  $\epsilon_1 = 0.05$ .

This constraint is computationally less expensive than (7), since only a subset of all the possible combinations of pairs of points, satisfying the ordering constraint, is considered.

Similarly, a second constraint can be derived for the angle of rotation  $\theta$ . Let  $\theta_{i,j}$  represent the angle of rotation that curve  $C$  undergoes to become  $C'$ , estimated from  $P_i, P'_i, P_j$  and  $P'_j$ . Because of the rigidity assumption, this angle is the same for all pairs of corresponding points. Thus, the second consistency constraint can be written as follows:

$$\forall i \in \{1..n-2\}, \quad |\theta_{i,i+1} - \theta_{i+1,i+2}| \leq \epsilon_2 \quad (9)$$

where  $\epsilon_2$  represents the maximum difference allowed in the angle of rotation. Typically  $\epsilon_2 = 5^\circ$ .

The two constraints above (8) and (9) not only reduce the complexity of the process by a great deal, but also categorically discard wrong points (*i.e.* those which constitute noise and digitisation effect), since such points would violate the constraints.

Further, such correspondence should be “loose” at both ends of each boundary, that is, it should allow genuine points at boundary ends not to be matched. Such points may be present due to occlusion.

Consequently, the desired matching is the longest one (*i.e.*, the one with the maximum number of corresponding points) satisfying all the above constraints. Theoretically, such a matching exists and is unique.

The parameters  $k$  and  $\theta$  are calculated respectively from  $k_{i,i+1}$ , and  $\theta_{i,i+1}$  of the final matching as follows:

$$k = \frac{1}{n-1} \sum_1^{n-1} k_{i,i+1} \quad \text{and} \quad \theta = \frac{1}{n-1} \sum_1^{n-1} \theta_{i,i+1} \quad (10)$$

## 6 Generalisation

In order to find the six unknown coefficients  $a^{(i)}$ ,  $i \in \{0..5\}$  of the general affine model given in (1), three pairs of points have to be considered.

If  $C$  is a boundary undergoing such transformation, then the parameters  $a^{(i)}$ ,  $i \in \{0..5\}$ , must be the same for any three pairs of corresponding points. Let  $P_1, P_2, \dots, P_n$  be the ordered peak points on  $C$ , and let  $P'_1, P'_2, \dots, P'_n$  be their corresponding peak points on

$C'$ . Let  $a_{i,j,k}^{(l)}$  be the coefficients estimated from points  $P_i, P'_i, P_j, P'_j, P_k$  and  $P'_k$ , by solving the system of six linear equations obtained from these points. The constraint on  $a_{i,j,k}^{(l)}$  can be expressed as follows:

$$\forall l \in \{0..5\}, \forall i \in \{1..n-3\},$$

$$|a_{i,i+1,i+2}^{(l)} - a_{i+1,i+2,i+3}^{(l)}| \leq \epsilon_l \quad (11)$$

where  $\epsilon_l$  is a small number representing the maximum differences allowed for  $a^{(l)}$   $l \in \{0..5\}$ .

The parameters are computed from the longest pairings with the maximum number of corresponding points, satisfying all the constraints of (11). Note that not all the constraints are needed: depending on the context, some constraints can be dropped. As in the restricted model, the longest pairings exists and is unique. The final value of the coefficient  $a^{(l)}$  is defined to be:

$$a^{(l)} = \frac{1}{n-2} \sum_1^{n-2} a_{i,i+1,i+2}^{(l)} \quad \forall l \in \{0..5\} \quad (12)$$

Unlike the restricted model presented earlier, the coefficients  $a^{(i)}$ ,  $i \in \{0..5\}$ , have no direct intuitive interpretations. This renders the choice of the differences allowed  $\epsilon_i$ ,  $i \in \{0..5\}$ , more empirical. Additionally, solving the linear system may turn out to be costly. In the restricted model, a linear system of four equations and four unknowns ( $k, \theta, x_0, y_0$ ) was implicitly solved in equations (3)–(5).

## 7 Constrained Point Matching

Curvature profiles are computed from cubic Spline approximations of each curve. However, it is not necessary to compute curvature. Curvature-like measurements can be used to compute locations of points of maximum curvature as in [8]. These dominant points are stored in arrays  $R$  and  $R'$  for processing.

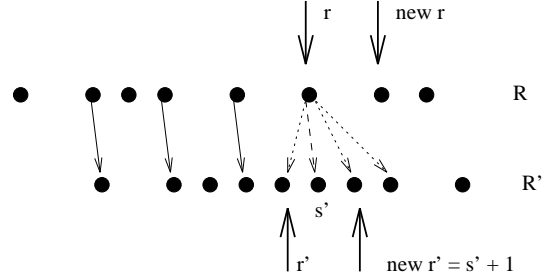


Figure 5. *Establishing correspondence between points. Point  $r$  is currently under examination. Among the  $M$  possible pairings (marked with dotted arrows),  $s'$  (marked with a dashed arrow) is the pairing consistent with the previously established pairs (marked with solid arrows). The search continues with the point marked new  $r$  under focus, and with  $r' = s' + 1$ .*

Note that at least two significant points must be present in  $C$  and in  $C'$ . Curves which do not satisfy this condition are ignored.

To find the longest match satisfying the above constraints, an exhaustive search is employed. Progressing from left to right, points from  $R$  and  $R'$  are tested in the following manner: let  $L = \{(p, p')\}$  be the list of points matched at a given stage; by construction, pairs in  $L$  satisfy the above conditions. Initially  $L = \emptyset$ . Let  $r$  and  $r'$  be the indices of the current points of  $R$  and  $R'$  under focus (see **Figure 5**). For all points between  $r'$  and  $r' + M$ , select  $s'$  such that the new pair  $(r, s')$  is consistent with all existing pairs in  $L$ . If such point exists, add  $(r, s')$  to  $L$ . If  $\#L > \#B$ , (where  $B$  is the previously best recorded match, and  $\#B$  its size), record  $L$  as the new best match, and continue with  $r = r + 1$  and  $r' = s' + 1$ . If, however, no point between  $r'$  and  $r' + M$  can be consistently paired with  $r$ , continue with  $r = r + 1$ .

The number of potential points from  $R'$  under scrutiny each time is a global parameter of the process, typically  $M = 3$ . The algorithm is intuitively recursive [12]. Its iterative version is as follows:

$Search(R, n, r, R', n', r', L, l)$

ARRAYS of POINTS R,R'  
 LENGTHS n,n',l  
 INDICES r,r'  
 LIST of PAIRS L  
 STRUCTURE {r, r', L, l} S

```

BEGIN
  WHILE (r ≤ n AND r' ≤ n') DO
    FOR i=r' TO r'+N DO
      IF Consistent(R[r],R'[i],L,l) THEN
        PUSH(Stack, S)
        L ← L ∪ {(R[r],R'[i])}
        r ← r+1, r' ← i+1, l ← l+1
      END
    END
  END
  WHILE ( NOTEMPTY(Stack) ) DO
    POP (Stack, S)
    IF Improvement(L, B) THEN
      B = L
    END
  END
END

```

Algorithm 7.1. *Constrained Point Matching Algorithm*

$Improvement(L, B)$  determines whether there is an improvement in the pairings of  $L$  with respect to the best recorded pairings at the time of the call. Clearly, if  $L$  contains more pairings than  $B$ , an improvement has occurred. If, however, the lists  $B$  and  $L$  have the same number of pairs,  $L$  is said to be better than  $B$  if the pairings in  $L$  give a smaller rotation angle than the pairings in  $B$ . This only happens when there is an intrinsic ambiguity in the transform.

The function  $Consistent(P, P', L, l)$  returns 1 if the pair  $(P, P')$  is consistent with the list of pairings  $L$  (of length  $l$ ) at the time of the call. The pair  $(P, P')$  is said to be consistent with  $L$  if the two constraints on the expansion factor and the angle of rotation are satisfied for the list  $L \cup \{(P, P')\}$ .

## 8 Evaluating the Method

The length of the longest pairings obtained is a good indication of the correctness of computed transformation; if this number is not large enough, it can be easily deduced that the parameters of the transform could not be correctly computed. This is the case for straight lines. In 2-D recognition, if a data curve and a model curve have enough points of significant curvature, but the number of pairings between them is low, a decision can be taken to reject the curve as part of a model object. In motion context, a low number of pairings implies that the model used for computing motion does not

fit the physical changes in the scene. In fact, the ratio  $r = \frac{p}{n}$ , where  $p$  is the number of paired points, and  $n$  is the smallest of number of points of the two curves, is a self-verifying indicator of the goodness of the model. The computed transform parameters are taken to be reliable if  $r \geq 0.5$ , which implies that 50% of the points of significant curvature are correctly matched in accordance with the constraints.

## 9 Results

In this section, we present results of the computation of the parameters of the transform. Comparisons are drawn with Ma and Chen's technique, which was actually implemented for this purpose, on both artificial and real data.

Boundary	Method	$k$	$\theta$	$(x_0, y_0)$
(a)	HT	1.0	90°	(52,2)
	AC	2.0	0°	(10,10)
	CPM	2.0	0°	(10,10)
(b)	HT	0.99	0°	(6,15)
	AC	1.0	8°	(5,10)
	CPM	1.009	8°	(5,11)
(c)	HT	1.0	-9°	(-2,1)
	AC	2.3	10°	(-5,10)
	CPM	2.27	10°	(-4,10)
(d)	HT	0.98	60°	(19,12)
	AC	2.0	4°	(-5,10)
	CPM	1.98	4°	(-5,10)

Table 1. *The parameters computed by the Hough Transform method (HT) and by our method: Constrained Point Matching (CPM). (AC) are the actual parameters. See Figure 6 for curves (a), (b), (c) and (d).*

Judging from the results displayed in [3], our technique performs at least as well as Cohen *et al.*'s for the particular case of boundaries undergoing *similarity* transformation (uniform deformation). It is also more conservative, for their technique is iterative and is based on a first guess.

Figure 6 (c) and (d) are designed to show that our technique accounts properly for occlusion at end points (left end of large curves). The parameters found are the correct parameters which would transform one boundary to the other regardless of their differences at end points. Compare these results with those obtained with [6]'s method (see Table 1).

In the following set of real experiments, boundaries are first extracted from sequences of images [10], and their correspondence is established [9]. Due to the physical setting (conveyor belt) or to the nature of small movements, boundary motion is assumed to follow the

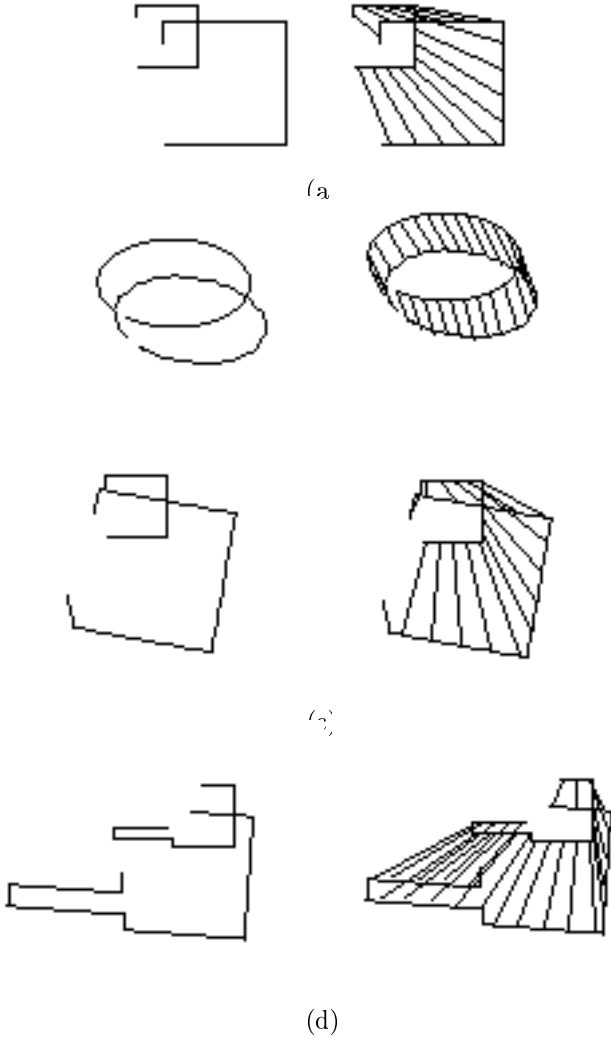


Figure 6. (a), (b), (c) and (d) are a set of artificial boundaries undergoing controlled transformations (see in **Table 1**). Experiments (c) and (d) in particular, show the ability of our technique to deal with occlusion at end points (see text).

(rotation, scale and translation) transform model, which is verified *a posteriori*, using the ration  $r$ .

**Figure 8** shows the velocities obtained for the two successive frames of **Figure 7** obtained from egomotion.

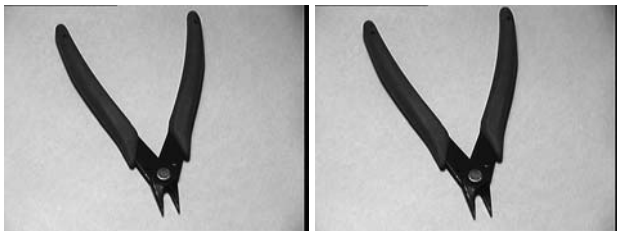


Figure 7. The image on the right was obtained by moving the camera forward. This movement follows the affine transform model.



Figure 8. Mapping obtained for the two-frame sequence of **Figure 7**, in which the wire snips undergo a small expansion, because of the forward motion of the camera.

**Figure 11** shows the velocities obtained by our approach and the Hough Transform approach for the two successive frames of **Figure 9**.

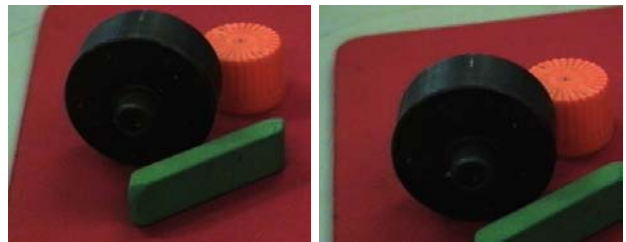


Figure 9. A simulated conveyor belt sequence: As the mouse pad is manually moved, the different objects undergoing a 3-D translation towards the camera. This motion is assumed to follow the Affine transform model.

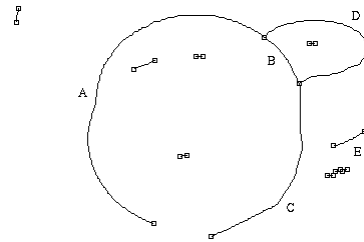


Figure 10. Boundaries extracted from the left image of the sequence of **Figure 9**.

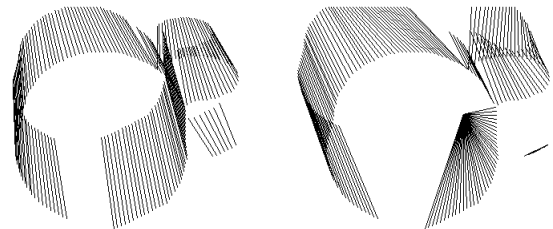


Figure 11. Left: the motions obtained by our technique. Right: the motions obtained with the Hough Transform approach for the sequence of frames in **Figure 9**.

Boundary	Method	$k$	$\theta$	$(x_0, y_0)$
Bdy A	HT	0.79	$0^\circ$	(-73,-77)
	CPM	0.80	$0^\circ$	(-39,-63)
Bdy B	HT	0.84	$-1^\circ$	(-24,-48)
	CPM	0.83	$0^\circ$	(-26,54)
Bdy C	HT	0.01	$180^\circ$	(-41,11)
	CPM	0.87	$0^\circ$	(-27,-95)
Bdy D	HT	0.98	$41^\circ$	(5,-105)
	CPM	0.66	$0^\circ$	(1,-55)
Bdy E	HT	0.0	$0^\circ$	(0,0)
	CPM	1.33	$0^\circ$	(-21,-53)

Table 2. *The parameters computed by the Hough Transform method (HT) and by our method: Constrained Point Matching (CPM). See Figure 10 for labels A, B, C, D and E.*

In this sequence, the objects undergo a translation towards the camera, as a result of the forward motion of the mouse pad (to simulate a conveyor belt). **Table 2** gives the values of the parameters obtained for the boundaries of the reverse sequence by the two methods. It can be seen clearly that the Hough Transform method *i.e.*, Ma and Chen’s, fails to find plausible parameters for boundary *C*. This is because of the occlusion effect, which our technique was designed to overcome. Also, the Hough Transform technique fails to find the correct parameters for boundary *D*. This, however, is due to the sub-optimality in curvature values computed by a crude method. Our technique, on the other hand, delivers correct parameters because curvature values are not used, but rather we use the locations of high curvature points. As expected, boundaries *A*, *B* and *C* all have similar expansion factors (0.80, 0.83 and 0.87). Boundary *D* on the other hand has a different expansion factor 0.66 which actually underlies the difference in depth with boundaries *A*, *B* and *C*. The parameters found for boundary *E*, however, are erroneous. This is because there is not a sufficient number of high curvature points to capture the motion. Note that Ma and Chen’s technique failed too.

## Conclusion

The various experiments demonstrate the robustness of our approach for estimating the parameters of boundary motion. This robustness stems firstly, from the formulation of the parameters of motion in terms of high curvature points locations, rather than curvature values.

Secondly, the algorithm developed to establish correspondence between points of significant curvature integrates local motions to establish a coherent global motion. This technique has been adapted to compute axes of symmetry of image objects [8].

Because of its sequential nature, the algorithm is rather sensitive to the first few correspondences found - they will all be consistent with the initial empty set - especially because of the occlusion problems at the ends of the fragments. This increases time complexity of the algorithm. A parallel constraint propagation algorithm working simultaneously from all the points of curvature would overcome this problem.

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