

Mth3301
 Homework #4 (Covers Modules 15-19)
 Due date to be set in class

1. (Hines, et al., 4-1). A refrigerator manufacturer subjects his finished products to a final inspection. Of interest are two categories of defects: scratches or flaws in the porcelain finish, and mechanical defects. The number of each type of defects is a random variable. The results of inspecting 50 refrigerators are shown in the following joint p.m.f. table, where X represents the occurrence of finish defects and Y represents the occurrence of mechanical defects.

$Y \setminus X$	0	1	2	3	4	5
0	11/50	4/50	2/50	1/50	1/50	1/50
1	8/50	3/50	2/50	1/50	1/50	
2	4/50	3/50	2/50	1/50		
3	3/50	1/50				
4	1/50					

- (a) Find the marginal distributions of X and Y .
- (b) Find the marginal distribution of mechanical defects, given that there are no finish defects.
- (c) Find the marginal distribution of finish defects, given that there are no mechanical defects.
2. (Hines, et al., 4-4). Consider a situation in which the surface tension and acidity of a chemical product are measured. These variables are coded such that surface tension is measured on a scale $0 \leq X \leq 2$, and acidity is measured on a scale $2 \leq Y \leq 4$. The probability density function of (X, Y) is

$$f(x, y) = \begin{cases} k(6 - x - y) & \text{if } 0 \leq x \leq 2, 2 \leq y \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find the appropriate value of k .
- (b) Calculate the probability that $X < 1, Y < 3$.
- (c) Calculate the probability that $X + Y \leq 4$.
- (d) Find the probability that $X < 1.5$.
- (e) Find the marginal densities of both X and Y .
3. Suppose that $f(x, y) = cxy^2$ for $0 < x < y^2 < 1$ and $0 < y < 1$.

- (a) Find c .
 - (b) Find the marginal p.d.f. of X , $f_X(x)$.
 - (c) Find the marginal p.d.f. of Y , $f_Y(y)$.
 - (d) Find $\mathbf{E}[X]$.
 - (e) Find $\mathbf{E}[Y]$.
 - (f) Find the conditional p.d.f. of X given $Y = y$, $f(x|y)$.
 - (g) Find the conditional expectation, $E[X|y]$.
 - (h) Find the “double” conditional expectation, $\mathbf{E}[\mathbf{E}[X|Y]]$.
4. (Hines, et al., 4–8.) Consider the probability distribution of the discrete random vector (X, Y) , where X represents the number of orders for aspirin in August in the neighborhood drugstore and Y represents the number of orders in September. The joint distribution is shown in the following table.

$Y \backslash X$	51	52	53	54	55
51	0.06	0.05	0.05	0.01	0.01
52	0.07	0.05	0.01	0.01	0.01
53	0.05	0.10	0.10	0.05	0.05
54	0.05	0.02	0.01	0.01	0.03
55	0.05	0.06	0.05	0.01	0.03

- (a) Find the marginal distributions.
 - (b) Find the expected sales in September, given that sales in August were either 51, 52, 53, 54, or 55, respectively.
5. (Hines, et al., 4–9). Assume that X and Y are coded scores of two intelligence tests, and the p.d.f. of (X, Y) is given by

$$f(x, y) = \begin{cases} 6x^2y & \text{if } 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the expected value of the score on test #2 given the score on test #1. Then find the expected value of the score on test #1 given the score on test #2.

6. (Hines, et al., 4–31). Given the following joint p.d.f.’s, determine whether or not X and Y are independent.
- (a) $g(x, y) = 4xye^{-(x^2+y^2)}$, $x > 0$, $y > 0$.

(b) $f(x, y) = 3x^2y^{-3}$, $0 < x < y < 1$.

(c) $f(x, y) = 6(1 + x + y)^{-4}$, $x > 0$, $y > 0$.

7. (Hines, et al., 4-19). Let X and Y have joint p.d.f. $f(x, y) = 2$, $0 < x < y < 1$. Find the correlation between X and Y .
8. (Hines, et al., 4-21). Consider the data in Problem 1 (Hines, et al., 4-1) above. Are X and Y independent? Find the correlation.
9. Let $\text{Var}(X) = \text{Var}(Y) = 20$, $\text{Var}(Z) = 30$, $\text{Cov}(X, Y) = 2$, $\text{Cov}(X, Z) = -3$, and $\text{Cov}(Y, Z) = -4$. Find $\text{Corr}(X, Z)$ and $\text{Var}(X - 2Y + 5Z)$.
10. (Hines et al. 4-18 m.g.f.)