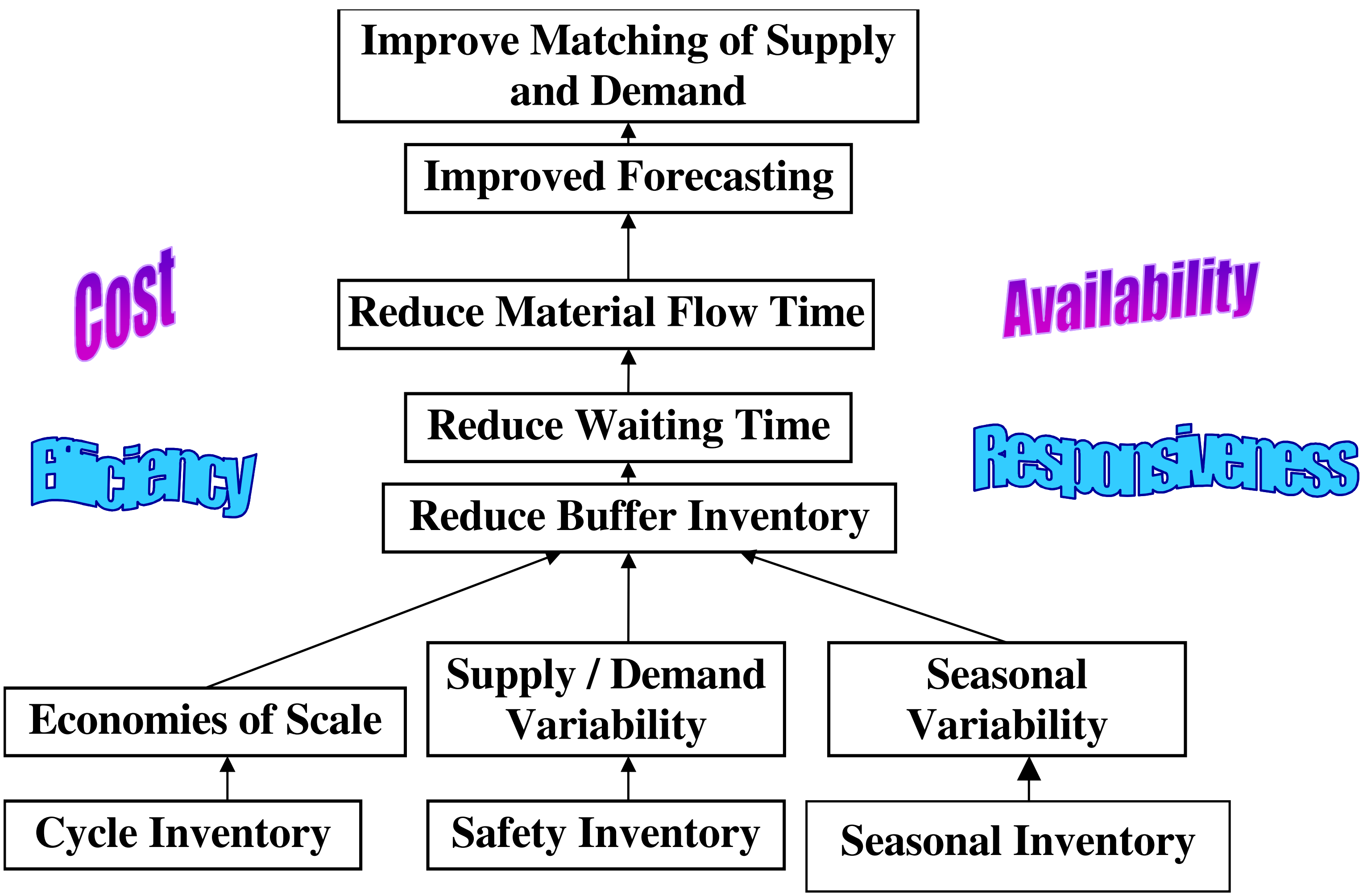
# Objectives of Chapters 10, 11

- Planning and Managing Inventories in a SC: (Ch10, 11, 12)
- Ch10→ Discusses factors that affect the level of cycle inventory within a SC and explain how it correlates to cost.
- Ch11→ focuses on the building and management of Safety Inventory to counter supply of demand uncertainty.
- Ch12→ Discusses factors that influence the appropriate level of product availability within a SC.

# Supply Chain Management (3rd Edition)

## Chapter 11
## Managing Uncertainty in the Supply Chain: Safety Inventory

# Role of Inventory in the Supply Chain



# Outline

- The role of safety inventory in a supply chain
- Determining the appropriate level of safety inventory
- Impact of supply uncertainty on safety inventory
- Impact of aggregation on safety inventory
- Impact of replenishment policies on safety inventory
- Managing safety inventory in a multi-echelon supply chain
- Estimating and managing safety inventory in practice

# The Role of Safety Inventory in a Supply Chain

- Forecasts are rarely completely accurate
- If average demand is 1000 units per week, then half the time actual demand will be greater than 1000, and half the time actual demand will be less than 1000; what happens when actual demand is greater than 1000?
- If you kept only enough inventory in stock to satisfy average demand, half the time you would run out
- Safety inventory: Inventory carried for the purpose of satisfying demand that exceeds the amount forecasted in a given period

# Role of Safety Inventory

- Average inventory is therefore cycle inventory plus safety inventory
- There is a fundamental tradeoff:
  - Raising the level of safety inventory provides higher levels of product availability and customer service
  - Raising the level of safety inventory also raises the level of average inventory and therefore increases holding costs
    - » Very important in high-tech or other industries where obsolescence is a significant risk (where the value of inventory, such as PCs, can drop in value)
    - » Compaq and Dell in PCs

# Two Questions to Answer in Planning Safety Inventory

- What is the appropriate level of safety inventory to carry?
- What actions can be taken to improve product availability while reducing safety inventory?

# Determining the Appropriate Level of Safety Inventory

- Measuring demand uncertainty
- Measuring product availability
- Replenishment policies
- Evaluating cycle service level and fill rate
- Evaluating safety level given desired cycle service level or fill rate
- Impact of required product availability and uncertainty on safety inventory

# Determining the Appropriate Level of Demand Uncertainty

- Appropriate level of safety inventory determined by:
  - supply or demand uncertainty
  - desired level of product availability
- Higher levels of uncertainty require higher levels of safety inventory given a particular desired level of product availability
- Higher levels of desired product availability require higher levels of safety inventory given a particular level of uncertainty

# Measuring Demand Uncertainty

- Demand has a systematic component and a random component
- The estimate of the random component is the measure of demand uncertainty
- Random component is usually estimated by the standard deviation of demand
- Notation:

  D = Average demand per period

  $\sigma_D$ = standard deviation of demand per period

  L = lead time = time between when an order is placed and when it is received
- Uncertainty of demand during lead time is what is important

# Measuring Demand Uncertainty

- Assume that demand during each of L periods is independent and normally distributed with a demand of D and standard deviation of $\sigma_D$ .
- P = demand during k periods = kD
- $\Omega$ = std dev of demand during k periods = $\sigma_D$Sqrt(k)
- Coefficient of variation = cv = $\sigma/\mu$ =(std dev) /mean= size of uncertainty relative to demand
- If demand during two successive period is correlated, then W is computed using equation (11.1).

# Measuring Product Availability

- Product availability: a firm's ability to fill a customer's order out of available inventory
- Stockout: a customer order arrives when product is not available
- Product fill rate (fr): fraction of demand that is satisfied from product in inventory (should me measured over specified amount of demand rather than time)

# Measuring Product Availability

- Order fill rate: fraction of orders that are filled from available inventory (Should be measured over a specified number of orders rather than time). Useful for multiproduct scenarios.
- Cycle service level (CSL): fraction of replenishment cycles that end with all customer demand met. CSL should be measured over a specified number of replenishment cycles. (A replenishment cycle is the interval between two successive replenishment deliveries.

# Replenishment Policies

- Replenishment policy: decisions regarding when to reorder and how much to reorder→ determines cycle and safety inventory + fr + CSL
- Several forms of replenishment policies including:
  - **Continuous review:** inventory is continuously monitored and an order of size Q is placed when the inventory level reaches the reorder point ROP→ Size of order does not change but time between orders may fluctuate.
  - **Periodic review:** inventory is checked at regular (periodic) intervals and an order is placed to raise the inventory to a specified threshold (the "order-up-to" level)→ Time between orders is fixed but size of order can fluctuate.

# Continuous Review Policy: Safety Inventory and Cycle Service Level

$L$: Lead time for replenishment

$D$: Average demand per unit time

$\sigma_D$: Standard deviation of demand per period

$D_L$: Mean demand during lead time

$\sigma_L$: Standard deviation of demand during lead time

$CSL$: Cycle service level

$ss$: Safety inventory

$ROP$: Reorder point

$$D_L = DL$$

$$\sigma_L = \sqrt{L}\,\sigma_D$$

$$ss = ROP - D_L$$

$$CSL = F(ROP, D_L, \sigma_L)$$

$$ss = F_S^{-1}(CSL) \times \sigma_L$$

*Average Inventory* = $Q/2 + ss$

# Example 11.1: Estimating Safety Inventory (Continuous Review Policy)

$D$ = 2,500/week; $\sigma_D$ = 500

$L$ = 2 weeks; $Q$ = 10,000; $ROP$ = 6,000

$D_L = DL = (2500)(2) = 5000$

$ss = ROP - D_L = 6000 - 5000 = 1000$

Cycle inventory = Q/2 = 10000/2 = 5000

Average Inventory = cycle inventory + ss = 5000 + 1000 = 6000

Average Flow Time = Avg inventory / throughput = 6000/2500 = 2.4 weeks

# Example 11.2: Estimating Cycle Service Level (Continuous Review Policy)

$D$ = 2,500/week; $\sigma_D$ = 500

$L$ = 2 weeks; $Q$ = 10,000; $ROP$ = 6,000

$$\sigma_L = \sigma_R\sqrt{L} = (500)\sqrt{2} = 707$$

Cycle service level, $CSL = F(D_L + ss, D_L, \sigma_L) =$

= NORMDIST ($D_L$ + ss, $D_L$, $\sigma_L$) = NORMDIST(6000,5000,707,1)

= 0.92 (This value can also be determined from a Normal probability distribution table)

# Fill Rate

- *Proportion of customer demand satisfied from stock*
- *Stockout occurs when the demand during lead time exceeds the reorder point*
- *ESC* is the expected shortage per cycle (average demand in excess of reorder point in each replenishment cycle)
- *ss* is the safety inventory
- *Q* is the order quantity

$$fr = 1 - \frac{ESC}{Q} = \frac{Q - ESC}{Q}$$

$$ESC = -ss\{1 - F_S\left(\frac{ss}{\sigma_L}\right)\} + \sigma_L f_S\left(\frac{ss}{\sigma_L}\right)$$

$ESC = -ss\{1\text{-NORMDIST}(ss/\sigma_L, 0, 1, 1)\} + \sigma_L \text{ NORMDIST}(ss/\sigma_L, 0, 1, 0)$

# Example 11.3: Evaluating Fill Rate

$ss$ = 1,000, $Q$ = 10,000, $\sigma_L$ = 707, Fill Rate ($fr$) = ?

$ESC$ = -$ss${1-NORMDIST($ss/\sigma_L$, 0, 1, 1)} +
$\sigma_L$ NORMDIST($ss/\sigma_L$, 0, 1, 0)
= -1,000{1-NORMDIST(1,000/707, 0, 1, 1)} +
707 NORMDIST(1,000/707, 0, 1, 0)
= 25.13

$fr$ = ($Q$ - $ESC$)/$Q$ = (10,000 - 25.13)/10,000 = 0.9975

# Factors Affecting Fill Rate

- Safety inventory: Fill rate increases if safety inventory is increased. This also increases the cycle service level.
- Lot size: For same safety inventory, fill rate increases on increasing the lot size even though cycle service level does not change.

# Example 11.4: Evaluating Safety Inventory Given CSL

$D$ = 2,500/week; $\sigma_{D}$ = 500

$L$ = 2 weeks; $Q$ = 10,000; $CSL$ = 0.90

$D_L$ = 5000, $\sigma_L$ = 707 (from earlier example)

ss = $F_S^{-1}(CSL)\sigma_L$ = [NORMSINV(0.90)](707) = 906

(this value can also be determined from a Normal probability distribution table)

$ROP = D_L + ss = 5000 + 906 = 5906$

# Evaluating Safety Inventory Given Desired Fill Rate

$D = 2500$, $\sigma_D = 500$, $Q = 10000$

If desired fill rate is $fr = 0.975$, how much safety inventory should be held?

$ESC = (1 - fr)Q = 250$

Solve

$$ESC = 250 = -ss\left[1 - F_S\left(\frac{ss}{\sigma_L}\right)\right] + \sigma_L f_S\left(\frac{ss}{\sigma_L}\right)$$

$$250 = -ss\left[1 - NORMSDIST\left(\frac{ss}{\sigma_L}\right)\right] + \sigma_L NORMDIST\left(\frac{ss}{\sigma_L},1,1,0\right)$$

# Evaluating Safety Inventory Given Fill Rate (try different values of ss)

| Fill Rate | Safety Inventory |
|---|---|
| 97.5% | 67 |
| 98.0% | 183 |
| 98.5% | 321 |
| 99.0% | 499 |
| 99.5% | 767 |

# Impact of Required Product Availability and Uncertainty on Safety Inventory

- Desired product availability (cycle service level or fill rate) increases, required safety inventory increases
- Demand uncertainty ($\sigma_L$) increases, required safety inventory increases
- Managerial levers to reduce safety inventory without reducing product availability
  - reduce supplier lead time, L (better relationships with suppliers)
  - reduce uncertainty in demand, $\sigma_L$ (better forecasts, better information collection and use)

# Impact of Supply Uncertainty on Safety Inventory

- $D$: Average demand per period
- $\sigma_{D:}$ Standard deviation of demand per period
- $L$: Average lead time
- $s_L$: Standard deviation of lead time

$$D_L = DL$$

$$\sigma_L = \sqrt{L\sigma_D^2 + D^2 s_L^2}$$

# Impact of Supply Uncertainty on Safety Inventory

$D$ = 2,500/day; $\sigma_{D\,=}$ 500

$L = 7$ days; $Q$ = 10,000; $CSL = 0.90$; $s_L = 7$ days

$D_L = DL = (2500)(7) = 17500$

$$\sigma_L = \sqrt{L\sigma_D^2 + D^2 s_L^2}$$

$$= \sqrt{(7)500^2 + (2500)^2(7)^2} = 17500$$

ss = $F^{-1}_s(CSL)\sigma_L$ = NORMSINV(0.90) x 17550

= 22,491

# Impact of Supply Uncertainty on Safety Inventory

Safety inventory when $s_L = 0$ is 1,695

Safety inventory when $s_L = 1$ is 3,625

Safety inventory when $s_L = 2$ is 6,628

Safety inventory when $s_L = 3$ is 9,760

Safety inventory when $s_L = 4$ is 12,927

Safety inventory when $s_L = 5$ is 16,109

Safety inventory when $s_L = 6$ is 19,298

# Impact of Aggregation on Safety Inventory

- Models of aggregation
- Information centralization
- Specialization
- Product substitution
- Component commonality
- Postponement

# Impact of Aggregation on Safety Inventory

$D_i$ : Mean weekly demand in region i, i=1,…, k

$\sigma_i$ : Standard deviation of weekly demand in region i

$\rho_{ij}$ : Correlation of weekly demand for regions i and j.

# Impact of Aggregation on Safety Inventory

- With no aggregation, the total safety inventory in decentralized option =

$$\sum_{i=1}^{k} F_s^{-1}(CSL) \times \sqrt{L} \times \sigma_i$$

# Impact of Aggregation on Safety Inventory

$$D^C = \sum_{i=1}^{n} D_i$$

$$\sigma_D^C = \sqrt{\sum_{i=1}^{n} \sigma_i^2 + 2 \times \sum_{i>j} \rho_{ij} \sigma_i \sigma_j}$$

If $\rho_{ij} = 0$ for all i, j then

$$\sigma_D^C = \sqrt{\sum_{i=1}^{n} \sigma_i^2} \text{ and } \sigma_L^C = \sqrt{L}\, \sigma_D^C$$

$$ss = F_s^{-1}(CSL) \times \sigma_L^C$$

# Impact of Aggregation on Safety Inventory

**Holding cost savings on aggregation per unit sold:**

$$\frac{F_s^{-1}(CSL) \times \sigma_L^C \times H}{D^c} \times \left(\sum_{i=1}^{k} \sigma_i - \sigma_D^C\right)$$

# Impact of Aggregation on Safety Inventory

- The safety inventory savings on aggregation increase with the desired CSL
- The safety inventory savings on aggregation increase with the L
- The safety inventory savings on aggregation increase with the H
- The safety inventory savings on aggregation decrease as the correlation coefficients increase

# Impact of Aggregation on Safety Inventory (Example 11.7)

Car Dealer : 4 dealership locations (disaggregated)

D = 25 cars; $\sigma_D$ = 5 cars; L = 2 weeks; desired CSL=0.90

What would the effect be on safety stock if the 4 outlets are consolidated into 1 large outlet (aggregated)?

<u>At each disaggregated outlet:</u>

For L = 2 weeks, $\sigma_L$ = 7.07 cars

ss = $F_s^{-1}(CSL) \times \sigma_L = F_s^{-1}(0.9) \times 7.07 = 9.06$

Each outlet must carry 9 cars as safety stock inventory, so safety inventory for the 4 outlets in total is (4)(9) = 36 cars

# Impact of Aggregation on Safety Inventory (Example 11.7)

One outlet (aggregated option):

$RC = D_1 + D_2 + D_3 + D_4 = 25+25+25+25 = 100$ cars/wk

$\sigma_R^C = Sqrt(5^2 + 5^2 + 5^2 + 5^2) = 10$

$\sigma_L^C = \sigma_D^C\ Sqrt(L) = (10)Sqrt(2) = (10)(1.414) = 14.14$

$ss = F_s^{-1}(CSL) \times \sigma_L^C = F_s^{-1}(0.9) \times 14.14 = 18.12$

or about 18 cars

If $\rho$ does not equal 0 (demand is not completely independent), the impact of aggregation is not as great (Table 11.3)

# Impact of Aggregation on Safety Inventory

- If number of independent stocking locations decreases by n, the expected level of safety inventory will be reduced by square root of n (*square root law*)
- Many e-commerce retailers attempt to take advantage of aggregation (Amazon) compared to bricks and mortar retailers (Borders)
- Aggregation has two major disadvantages:
  - Increase in response time to customer order
  - Increase in transportation cost to customer
  - Some e-commerce firms (such as Amazon) have reduced aggregation to mitigate these disadvantages

# Information Centralization

- Virtual aggregation
- Information system that allows access to current inventory records in all warehouses from each warehouse
- Most orders are filled from closest warehouse
- In case of a stockout, another warehouse can fill the order
- Better responsiveness, lower transportation cost, higher product availability, but reduced safety inventory
- Examples: McMaster-Carr, Gap, Wal-Mart

# Specialization

- Stock all items in each location or stock different items at different locations?
  - Different products may have different demands in different locations (e.g., snow shovels)
  - There can be benefits from aggregation
- Benefits of aggregation can be affected by:
  - coefficient of variation of demand (higher cv yields greater reduction in safety inventory from centralization)
  - value of item (high value items provide more benefits from centralization)
  - Table 11.4

# Value of Aggregation at Grainger (Table 11.4)

| | *Motors* | *Cleaner* |
|---|---|---|
| Mean demand | 20 | 1,000 |
| SD of demand | 40 | 100 |
| Disaggregate cv | 2 | 0.1 |
| Value/Unit | $500 | $30 |
| Disaggregate ss | $105,600,000 | $15,792,000 |
| Aggregate cv | 0.05 | 0.0025 |
| Aggregate ss | $2,632,000 | $394,770 |
| Holding Cost Saving | $25,742,000 | $3,849,308 |
| Saving / Unit | $7.74 | $0.046 |

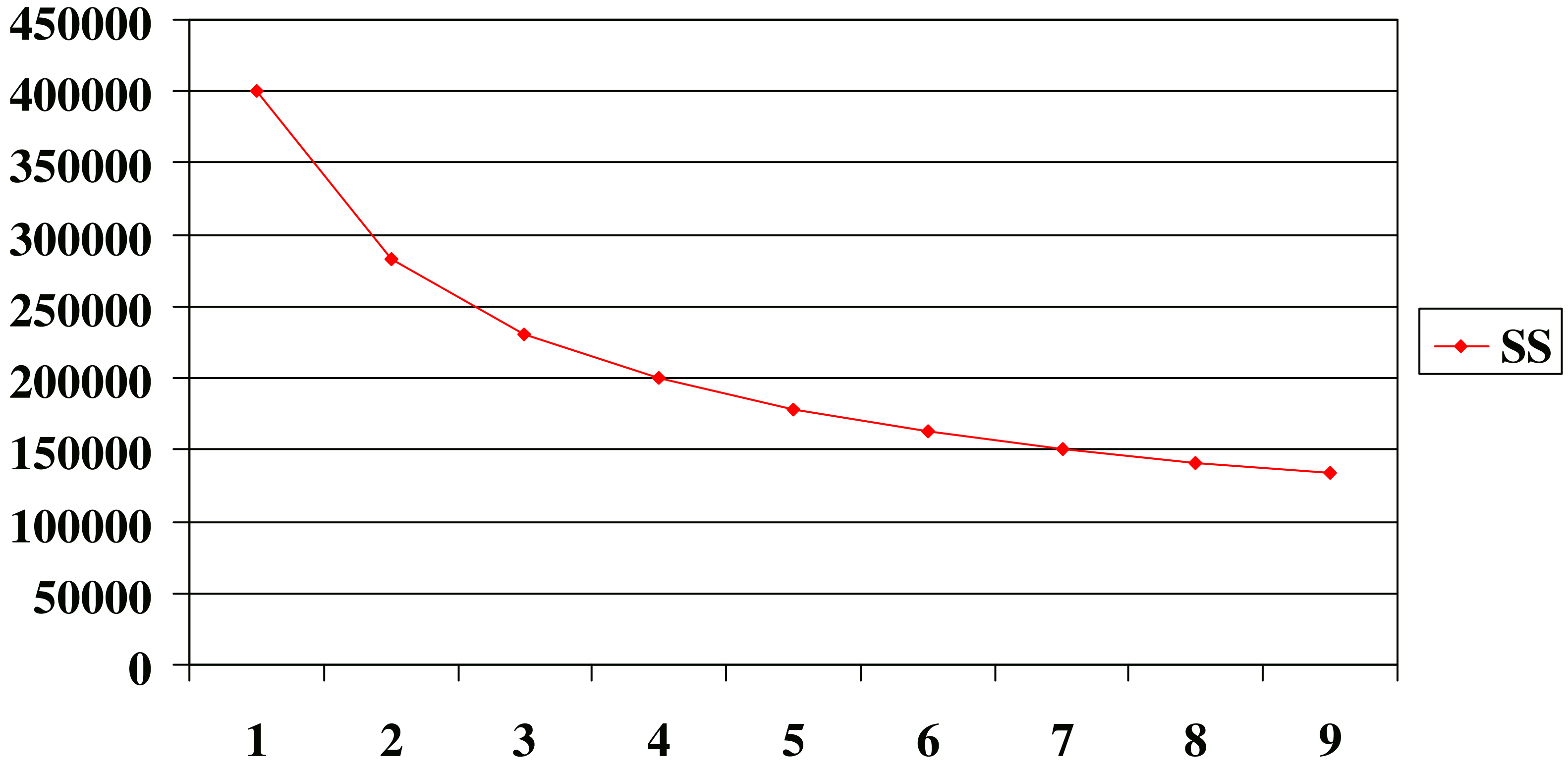
# Product Substitution

- Substitution: use of one product to satisfy the demand for another product
- Manufacturer-driven one-way substitution
- Customer-driven two-way substitution

# Component Commonality

- Using common components in a variety of different products
- Can be an effective approach to exploit aggregation and reduce component inventories

# Example 11.9: Value of Component Commonality



# Postponement

- The ability of a supply chain to delay product differentiation or customization until closer to the time the product is sold
- Goal is to have common components in the supply chain for most of the push phase and move product differentiation as close to the pull phase as possible
- Examples: Dell, Benetton

# Impact of Replenishment Policies on Safety Inventory

- Periodic review policies require more safety inventory than continuous review policies for the same level of product availability and lead time.
- Companies partition their products based on their value.
  - High value products are managed using continuous review policies
  - Low value products are managed using periodic review policies

# Estimating and Managing Safety Inventory in Practice

- Account for the fact that supply chain demand is lumpy
- Adjust inventory policies if demand is seasonal
- Use simulation to test inventory policies
- Start with a pilot
- Monitor service levels
- Focus on reducing safety inventories

# Summary of Learning Objectives

- What is the role of safety inventory in a supply chain?
- What are the factors that influence the required level of safety inventory?
- What are the different measures of product availability?
- What managerial levers are available to lower safety inventory and improve product availability?