Chapter 14: Effects of Inflation

Session 25, 26
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Topics to Be Covered in Today’s Lecture

Section 14.1: Impacts of Inflation;
Section 14.2: Present Worth with Inflation;
Section 14.3: Future Worth with Inflation;
Section 14.4: Capital Recovery Calculations Adjusted for Inflation.
14.1 Understanding the Impact of Inflation- Definition

- The increase in the amount of money necessary to obtain the same amount of product or service before the inflated price was present;
- Social Phenomena where too much money chases too few goods/services;
- Impact because the value of the currency changes downward in value-- It takes more dollars for the same amount of goods and services,
14.1 Understanding the Impact of Inflation- Definition

• Inflation rate is sensitive to real, as well as perceived factors of the economy.

• Factors such as energy, interest rates, availability and cost of skilled people, scarcity of materials, political stability, and other less tangible factors have short term and long term impact on the inflation rate.

• It is vital to take into account the effects of inflation in an economic analysis
14.1 Deflation

• Where the value of the currency increases over time and produces increased value;
• Less amounts of the currency can purchase more goods and services than before.
• Not commonly seen……any more!
14.1 Equating “Value”

- Money in time period $t_1$ can be related to money in time period $t_2$ by the following:

$$\text{Dollars}_{t_1} = \frac{\text{Dollars}_{t_2}}{\text{inflation rate between } t_1 \text{ and } t_2}$$

- Where $\text{Dollars}_{t_1}$ are today’s dollars or what we call constant value dollars.

- And $\text{Dollars}_{t_2}$ represent future dollars.
14.1 The Inflation rate - $f$

- The inflation rate, $f$, is a percent per time period;
- Similar to an interest rate.
- Let $n$ represent the period of time between $t_1$ and $t_2$ then . . . .
14.1 The Basic Inflation Relationship

- Future Dollars = Today’s dollars \( (1+f)^n \)
- Dollars in period \( t_1 \) are termed:
  - Constant-value or today’s dollars
- Dollars in time period \( t_2 \) are termed:
  - Future Dollars or,…
  - Then-current Dollars.
14.1 Critical Relationships to Remember

Constant-Value dollars = \( \frac{\text{future dollars}}{(1+f)^n} \)

Future dollars = today's dollars \((1+f)^n\).
14.1 Examples to Consider

- Assume a firm desires to purchase a productive asset that costs $209,000 in today’s dollars.
- Assume a reasonable inflation rate of say, 4% per year;
- In 10 years, that same piece of equipment would cost:
  - $209,000(1.04)^{10} = $309,371!
  - Does not count an interest rate or rate of return consideration.
14.1 Inflation can be Significant

- From the previous example we see that even at a modest 4% rate of inflation, the future impact on cost can and is significant!
- The previous example does not consider the time value of money.
- A proper engineering economy analysis should consider both inflation and the time value of money.
14.1 Three Important Rates

• Real or inflation-free interest rate.
  – Denoted as “\(i\)”.  

• Inflation-adjusted interest rate.
  – Denoted as “\(i_f\)”  

• Inflation rate.
  – Denoted as “\(f\)”.  

• In this chapter, \(i\), \(i_f\), and \(f\) are used extensively!
14.1 Real or Inflation-free Interest Rate - $i$

- Rate at which interest is earned;
- Effects of any inflation have been removed;
- Represents the actual or real gain received/charged on investments or borrowing.
14.1 Inflation-adjusted rate – $i_f$

- The interest rate that has been adjusted for inflation;
- Common Term –
  - *Market Interest Rate.*
  - Interest rate adjusted for inflation.
- The $i_f$ rate is the combination of the real interest rate – $i$, and the inflation rate – $f$.
- $i_f =$ function of $(i, f)$
- Also called the *inflated interest rate.*
14.1 Inflation rate - $f$

- The measure of the rate of change in the value of a currency.
- Similar to an interest rate but should not be confused as an interest rate.
- $f$ is a percentage value similar to the interest rate – $i$. 
Section 14.2 Present Worth and Inflation

- Prior chapters present worth was calculated assuming that all cash flows were in *constant value dollars*;
- Study Table 14-1 on page 475 for an example of $5,000 inflated at 4% per year with a discount rate of 10% per year.
## 14.2 Table 14-1 Analysis

<table>
<thead>
<tr>
<th>Inf. Rate</th>
<th>4.00%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Int. Rate</td>
<td>10.00%</td>
</tr>
<tr>
<td>P. Amt</td>
<td>$5,000.00</td>
</tr>
<tr>
<td>Year n</td>
<td>Cost inc. due to Inflat.</td>
</tr>
<tr>
<td>0</td>
<td>$5,000.00</td>
</tr>
<tr>
<td>1</td>
<td>$200</td>
</tr>
<tr>
<td>2</td>
<td>$208</td>
</tr>
<tr>
<td>3</td>
<td>$216</td>
</tr>
<tr>
<td>4</td>
<td>$225</td>
</tr>
</tbody>
</table>

$5,000 four years from now at a 4% inflation rate with a 10% discount rate is equivalent to $3415 at t = 0.
14.2 Derivation of a combine interest rate

• We now derive $i_f$ – the inflation adjusted interest rate.

• Start with:

$$P = F \frac{1}{(1+i)^n}$$

Assume $i$ is the real interest rate.
14.2 Derivation - continued

- Assume F is a future dollar amount with inflation built in. F should be converted to today’s dollars.

- P is then seen to be:

\[
P = F \frac{1}{(1+f)^n} \frac{1}{(1+i)^n}
\]

\[
P = F \frac{1}{(1+i+f+if)^n}
\]
14.2 Defining \( i_f \)

- \( i_f \) is then seen to equal:

\[
i_f = (i + f + i_f)
\]

Then,

\[
P = F \frac{1}{(1+i_f)^n} = F(P/F, i_f, n)
\]

Where \( F \) is expressed in future dollars
14.2 Example

• Assume $i = 10\%$ per year;
• $f = 4\%$ per year;
• $i_f$ is then calculated as:

\[ i_f = 0.10 + 0.04 + 0.10(0.04) = 0.144 = 14.4\% \text{/year} \]
14.2 Rework the $5000 example

• Apply the combined interest rate approach to validate Table 14-1.

• See the next slide!
14.2 Example Using the Combined Rate

<table>
<thead>
<tr>
<th>Year</th>
<th>Cost in Future Dollars</th>
<th>P/F,i(f),n</th>
<th>PW @ combined i-rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$5,000</td>
<td>1.0000</td>
<td>$5,000</td>
</tr>
<tr>
<td>1</td>
<td>$5,200</td>
<td>0.8741</td>
<td>$4,545</td>
</tr>
<tr>
<td>2</td>
<td>$5,408</td>
<td>0.7641</td>
<td>$4,132</td>
</tr>
<tr>
<td>3</td>
<td>$5,624</td>
<td>0.6679</td>
<td>$3,757</td>
</tr>
<tr>
<td>4</td>
<td>$5,849</td>
<td>0.5838</td>
<td>$3,415</td>
</tr>
</tbody>
</table>

Same Result as Table 14-1
14.2 Comparison of $$ Values
14.2  See Example 14.3

• Example 14.3 reinforces the concept of combining the discount rate with an assumed inflation rate.

• For example 14.2 the $i_f = 12\% / \text{year}$ and the discount rate – $i, = 15\% / \text{year}$.

• The $i_f$ for this example = $27.65\% / \text{year}$.

• See pages 479 – 480.
Section 14.3 Future Worth Analysis

- This section deals with solving:
  \[ F = f(i, i_f, n) \]

- There are four different interpretations for future worth calculations.
  1. Actual future $$;
  2. Purchasing power of future $$;
  3. Future $$ required at \( t = n \) to maintain \( t = 0 \) purchasing power;
  4. $$ at \( t = n \) to maintain purchasing power and earn a stated interest rate of \( i\% \) per time period.
14.3 Case 1: Actual $$$ at time $t = \"n\"$

- Solve:
  
  \[- F = P(1+i_f)^n = P(F/P,i_f,n) \]

- We apply the following equation for $i_f$:

\[
i_f = (i + f + i_f) \quad \text{[14.6]}\]
14.3 Case 1: Example

- P = $1,000 now and the market rate of interest is 10% per year. ($i_f = 4%/yr$).
- Remember, the market rate of interest includes both the inflation rate and the discount rate.
- If n = 7, what is the future value of the $1,000 now?
  \[ F = 1,000 \times (F/P, 10\%, 7) \approx 1,948. \]
14.3 Case 2: Constant-Value

1. Calculate $F_n$ using the market rate of interest.
2. $F_7 = $1000(F/P, 10%, 7) =$1948.
3. “Deflate” the $F_7$ dollars at the inflation rate.
4. $F = $1948/(1.04)^7$
   
   =1948/1.3159 = $1481;
   
   $1481$ is 24% less that the $1948$;
   
   Inflation reduces the purchasing power by 24% over the 7-year period.
14.3 Case 2: Equations

- Constant Value with Purchasing Power.

\[ F = \frac{P(1+i_f)^n}{(1+f)^n} \]

\[ F = \frac{P(F/P, i_f, n)}{(1+f)^n} \]
14.3 Finding the REAL interest Rate

• Given a market rate of interest;
• and, the inflation rate, f;
• Find the real interest rate.
• We know:
  – $i_f = i + f + if$
  – Solve for $i$:

$$i = \frac{i_f - f}{1 + f}$$
14.3 The Real Interest Rate

• The real rate, $i$:
• Is the rate at which current $$ expand with their same purchasing power;
• Into equivalent future $$.
• If “$f$” > Mkt. Rate then one will have a negative real interest rate.
14.3 Given $i_f = 10\%$ and $f = 4\%$

- Find the real interest rate for this case.
- Find the real interest rate for a market rate of 10\% and inflation at 4\%.
- Solve: 
  \[ i = \frac{0.10 - 0.04}{1 + 0.04} = 0.0577 = 5.77\% \]

\[ F = 1,000(F/P,5.77\%,7) = 1,481 \]

Inflation of 4\%/yr has reduced to real rate to less than 6\% per year!
14.3 Case 3: Future Amt. With No interest

- With inflation;
- Prices and costs increase over time;
- Future $$ are worth less out in time;
- At $t = n$, more $$ are needed;
- We only need to apply the inflation rate to the present sum;
- $F = P(1+f)^n$;
- $F = $1,000(1.04)^7 = $1316$
14.3 Case 4: Inflation and Real Interest

- Assume a firm has established their required MARR;

- Objective:
  - Maintain purchasing power and,
  - The time value of money.

- Approach:
  - Calculate $i_f$ and apply the required equivalence formulas at the $i_f$ rate.
14.3 Case 4: Apply the $i_f$ interest rate

- Given $P = $1,000
- $f = 4\%$
- Real interest rate of 5.77%;
- Calculate $i_f$ as;
  - $i_f = 0.0577 + 0.04 + 0.0577(0.04) = 0.10$
- Then:
  - $F = $1000(F/P,10\%,7) = $1948
14.3 Case 4: Apply the $i_f$ interest rate

• $i_f = 0.0577 + 0.04 + 0.0577(0.04) = 0.10$

• Then:
  – $F = 1000(F/P, 10\%, 7) = 1948$

• $1948$ seven years out is equivalent to $1,000$ now with a real return of $5.77\%$ per year and inflation at $4\%$ per year.
14.3 Setting the MARR rate (Sec. 10.4)

- Firms should set their MARR rate:
  - Cover the cost of capital;
  - Cover or buffer the inflationary aspects perceived to exist;
  - Account for risk.
14.3 Inflation adjusted MARR

• Let \( MARR_f \) = the inflation adjusted MARR;

• Then define the \( MARR_f \) as:
  \[ MARR_f = i + f + i(f) \]

• Thus;
  \[ F = P(1 + MARR_f)^n \]
  \[ F = P(F/P, MARR_f, n) \]
14.3 Importance of Inflation Impacts

• Most countries – inflation is from 2% to 8% per year;
• Some countries with weak currencies, political instability, poor balance of payments can have hyperinflation (as high as 100% per year).
14.3 Hyperinflation

- Spend money almost immediately;
- Loses value quickly;
- Very difficult to perform engineering economy calculations in a hyper inflated economy;
- Future values are unreliable and,
- Future availability of investment capital is very uncertain.
14.4 Capital Recovery Analysis Adjusted for Inflation

• With inflation present:
  – Current dollars invested in a productive asset must be recovered over time with future inflated dollars.
  – With loss of future purchasing power, future dollars will have less buying power than current dollars;
  – More dollars will be required to recover a present investment is a productive asset.
14.4 Capital Recovery Analysis - Inflation

- Assume an investment of $1000 today in a productive asset.
- Assume inflation is 8% per year and a real interest rate of 10% is required.
- Assume a 5-year recovery period and a 0 salvage value;
- \( A = 1000(A/P, 18.8\%, 5) = \$325.59/\text{year}. \)
  - Use the \( i_f \) formula to determine the 18.8% rate.
14.4 Capital Recovery Analysis - Inflation

- A = 1000(A/P, 18.8\%, 5) = $325.59/year (in future dollars).
- The annual equivalent of $1000 five years from now at 18.8\% is:
  - A = 1000(A/F, 18.8\%, 5) = $137.59/year.
- For F = $1000 at a real rate of 10\% (without inflation) is:
  - 1000(A/F, 10\%, 5) = $163.80/year.
14.4 Capital Recovery – inflation adjusted

- For a fixed “F” value;
- Uniformly distributed future costs should be spread over as long a period of time as possible;
  - The leveraging effect of inflation will reduce the payment to $137.59 vs. $163.80.
Chapter 14 Summary

- Inflation when treated like an interest rate makes the cost of the same product or service increase over time;
- This is due to the decreasing purchasing power of the currency when inflation is in effect.
Chapter 14 Summary cont.

• View in terms of:
  – Today’s dollars (constant-value);
  – Future dollars (then-current);

• Important Relationships:
  – Inflated interest rate;
    • \( i_f = i + f + if \)
  – Real Interest Rate:
    • \( i = (i_f - f)/(1+f) \)
Chapter 14 Summary cont.

- **PW of F with inflation:**
  \[ P = F\left(\frac{P}{F}, i_f, n\right). \]

- **Future worth of P in constant-value dollars with the same purchasing power:**
  \[ F = P\left(\frac{F}{P}, i, n\right). \]

- **F to cover a current amount with no interest:**
  \[ F = P\left(\frac{F}{P}, f, n\right) \]
Chapter 14 Summary cont.

• Hyperinflation:
  – Very high “f” values;
  – Available funds are expended immediately;
  – Because of increasing costs due to a rapid loss of purchasing power.
Chapter 14 Summary cont.

• F to cover a current amount with interest:
  – \( F = P(F/P, i_f, n) \).

• Annual Equivalent of a future dollar amount:
  – \( A = F(A/F, i_f, n) \).

• Annual equivalent of a present amount in future dollars:
  – \( A = P(A/P, i_f, n) \).
Assignments and Announcements

- Assignments due at the beginning of next class:
  - Online Quizzes for chapter 14 Due