Chapter 6:
ANNUAL WORTH ANALYSIS

Session 15
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Topics to Be Covered in Today’s Lecture

- Section 6.1: Advantages and uses of Annual Worth
- Section 6.2: Capital Recovery and AW Values
- Section 6.3 Evaluating Alternatives by Annual Worth Analysis
- Section 6.4 AW of a Perpetual Investment
Section 6.1: Advantages and uses of Annual Worth

• Popular Analysis Technique
• Easily understood-results are reported in $/time period
• Eliminates the LCM problem associated with the present worth method
  – Only have to evaluate one life cycle of a project
6.1 AW Calculations

• General in nature such that:
  – $AW = PW(A/P, i\%, n)$
  – $AW = FW(A/F, i\%, n)$

• Convert all cash flows to their end of period equivalent amounts
6.1 EQUIVALENT ANNUAL COST

- Cash Flow analysis approach where the cash flows are converted to their respective equal, end-of-period amounts.
- The result is reported in terms of $/period
- Variant of the present worth approach
- Popular with some managers who tend to think in terms of “$/year, $/months, etc
6.1 AW and Repeatability Assumption

• If two or more alternatives possess unequal lives then one need only evaluate the AW for any given cycle

• The annual worth of one cycle is the same as the annual worth of the other cycles (by assumption)
6.1 Repeatability Assumption

- Given alternatives with unequal lives
- The assumptions are:

1. The services so provided are needed forever
2. The first cycle of cash flows is repeated for successive cycles
3. All cash flows will have the same estimated values in every life cycle
6.1. One or More Cycles

Find the annual worth of any given cycle ($/period)

Annualize any one of the cycles

AW assumes repeatability of CF’s
6.1 6 year & 9 year Problem (Ex 6.1)

- See Example 6.1

- Present Worth would mandate a 18-year study period
  - 3 Cycles of the 6 year project
  - 2 cycles of the 9 year project
6.1 6 year & 9 year Problem (Ex 6.1)

- Need an 18 year study period for both projects.
- Present Worth would mandate a 18-year study period
  - 3 Cycles of the 6 year project
  - 2 cycles of the 9 year project
- Means a lot of calculation time!
6.1 Example 6.1 continued

• If one assumes the cash flow patterns remain the same for the 6 and 9 year projects then all one has to do is:

- Find the AW of any 6 – year cycle
- Find the annual worth of any 9-year cycle

And then compare the $AW_{6}/yr$ to $AW_{9}/yr$
6.1 Example 6.1 continued

• The Cash Flow Diagrams are:

![Cash Flow Diagram]

**Using the three cycles:**

\[
PWA = -15,000 - 15,000(P/F,15\%,6) + 1000(P/F,15\%,6) - 15,000(P/F,15\%,12) + 1000(P/F,15\%,12) + 1000(P/F,15\%,18) - 3500(P/A,15\%,18) = \$-45,036
\]

\[
AW = -45,036(A/P, 15\%, 18) = \$7349
\]

**Using one cycle:**

\[
AW = -15,000(A/P, 15\%, 6) + 1,000(A/F,15\%,6) - 3500 = \$7349
\]
6.1 Example

• Consider a project with $3000 annual operating cost and a $5000 investment required each 5 years. i = 10%

For one cycle

\[ EAC = 3000 + 5000(A/P, 10\%, 5) = \$4319/yr \]
6.1 Multiple cycle..same result!

For two cycles

\[
EAC = 3000 + 5000 \times (1+(P|F, .10, 5))(A|P, .10, 10)
\]

\[
= 3000 + 1319 = \mathbf{$4319/yr}$
\]
6.1 Advantages of AW

• Applicable to a variety of engineering economy studies
  – Asset Replacement and retention time studies to minimize overall annual costs (Chap11)
  – Breakeven Analysis and Make or Buy Decisions (Chap13)
  – Studies dealing with mfg. Costs
  – Economic Value Added analysis (EVA™)
6.1 AW Requirements

• Similar to the Present Worth Method, AW analysis requires:
  – A discount rate before the analysis is started
  – Estimates of the future cash flows
  – Estimate of the time period(s) involved
Section 6.2: Calculation of Capital Recovery and AW Values

• Assume the potential purchase of any productive asset

• One needs to know or estimate:
  – Initial Investment - P
  – Estimated Future Salvage Value - S
  – Estimated life of the asset - N
  – Estimated operating costs and timing- A
  – Operative interest rate – i% (Usually the MARR)
6.2 CAPITAL RECOVERY COST

• Thus, management is concerned about the equivalent annual cost of owning a productive asset.

• This cost is termed “Capital Recovery” Cost

• CR is a function of \{P, S, i\%, and “n” \}
6.2 CAPITAL RECOVERY COST (CR)

• CR = the equivalent annual worth of the asset given:

\[
\text{Capital Recovery (CR) is the annualized equivalent of the initial investment, } P_0 \text{ and the annualized amount of the future salvage value } F_n
\]
6.2 CAPITAL RECOVERY COST- CR

- **Given:**
- **Convert to:**

$A \text{ per year (CR)}$
6.2 CAPITAL RECOVERY COST

Cost = “−” and SV = “+” by convention

• COMPUTING CR FOR INVESTMENTS WITH SALVAGE VALUES:

\[ CR = -\left[ P(A|P, i, n) - S(A|F, i, n) \right] \]
6.2 CAPITAL RECOVERY COST

- COMPUTING CR FOR INVESTMENTS WITH SALVAGE VALUES:

- Method I - compute EAC of the original cost and subtract the EAC of the salvage value

- \[ CR = -[P(A|P, i, n) - S(A|F, i, n)] \]
6.2 More Traditional CR Approach

• Method II - Subtract the salvage value from the original cost and compute the annual cost of the difference. Add to that the interest which the salvage value would return each year, SV (i).

• \[
    CR(i\%) = -[(P - S) (A|P, i, n) + S(i)]
    \]
6.2 CR - Explained

• \[ CR(i\%) = -[(P - SV) (A|P, i, n) + SV(i)] \]

(Because \((A|F, i, n) = (A|P, i, n) -i\))

\textit{CR is the annual cost associated with owning a productive asset over N time periods at interest rate “i\%” per period}
6.2 CR –Sign Conventions

• Two approaches for signs
  – Let “P” carry a negative sign (investment)
  – Let “S” carry a positive sign (if a positive salvage value)
  – Then CR will carry a negative sign

• Let “P” carry a positive sign and “S” a negative sign
  – Treats “costs” as positive values and any positive salvage as a negative value
Section 6.3 Evaluating Alternatives by Annual Worth Analysis

• Given a discount rate (in advance);
• AW is perhaps the easiest method to apply for analysis of alternatives
• Mutually Exclusive Analysis
  – Select the one best alternative (Service vs. Revenue)
• Single Alternative
  – Accept if AW positive (MARR is met or exceeded), else reject
6.3 AW – Mutually Exclusive

• Given a set of two or more alternatives determine the AW at i\% then
  – Select the alternative with the lowest annual cost or the highest annual net cash flow
  – If pure cost situation – select min cost alternative
  – If mixed costs and revenues – select the max AW(i\%) alternative
6.3 Example 6.3

- Cash Flow Diagram is:

\[ P = -23,000 \]

\[ S = +$1500 \]

\[ A = +$1200/\text{yr} \]

\[ -$650 \quad -$700 \quad -$750 \quad -$800 \quad -$850 \]
6.3 Example 6.3

The Capital Recovery component is:

\[
S = +$1500
\]

\[
P = -23,000
\]

\[
CR(10\%) = -23,000(A/P, 10\%, 5) + 1500(A/F, 10\%, 5) = -$5822
\]
6.3 Example 6.3

- (Revenue – Operating Costs) are:

\[ A = \pm \$1200/yr \]

<table>
<thead>
<tr>
<th>Year</th>
<th>Revenue</th>
<th>Operating Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1200</td>
<td>$650</td>
</tr>
<tr>
<td>2</td>
<td>$700</td>
<td>$700</td>
</tr>
<tr>
<td>3</td>
<td>$750</td>
<td>$750</td>
</tr>
<tr>
<td>4</td>
<td>$800</td>
<td>$800</td>
</tr>
<tr>
<td>5</td>
<td>$850</td>
<td>$850</td>
</tr>
</tbody>
</table>
6.3 Example 6.3

- Cost/Revenue component is seen to equal:

\[
1.8101 \\
= +550 -50(A/G,10\%,5) \\
= 550 - 90.50 \\
= $459.50
\]
6.3 Example 6.3

- **Total Annual worth (CR + Cost/Rev)**
  - CR(10%) = -$5822
  - Revenue/Cost Annual amount: $459.50
  - AW(10%) = -$5822+$459.50
  - AW(10%) = -$5,362.50

This amount would be required to recover the investment and operating costs at the 10% rate on a per year basis.
6.3 What if one of the assumptions is not verified?

• A study period analysis should then take place:
  – Impress a study period
  – The cash flow over the study period are converted to annual amounts.
  – Read example 6.4
Section 6.4 AW of a Perpetual Investment

• EAC of a perpetual investment

• If an investment has no finite cycle it is called a **perpetual investment**. If “P” is the present worth of the cost of that investment, then EAC is P times i, the interest P would have earned each year.

\[ \text{EAC} = A = P(i) \]

**Remember:** \( P = A/i \)

*From the previous chapter*
6.4 Example: Perpetual Investment

• EXAMPLE

Two alternatives are considered for covering a football field. The first is to plant natural grass and the second is to install AstroTurf. Interest rate is 10%/year. Assume the field is to last a “long time”.

6.4 Example: Continued

Alternative A:

Natural Grass - Replanting will be required each 10 years at a cost of $10,000. Annual cost for maintenance is $5,000. Equipment must be purchased for $50,000 which will be replaced after 5 years with a salvage value of $5,000

Alternative B:

AstroTurf: Initial investment of $150,000 and $5,000/year maintenance costs
6.4 – Example: Natural Grass

Since cost is predominate, let (+) = cost and (-) = salvage values

Alternative A:

A = $5,000

P = $50,000 + $10,000

F_5 = $5,000

F_5 = $5,000

P = $50,000 + $10,000

F_5 = $50,000
6.4 Example: Natural Grass: Analysis

- (+) $60,000(A/P,10\%,10)
- (+) $5,000 (already an annual cost)
- (+) $50,000(P/F,10\%,5)(A/P,10\%,10)
- (-) $5,000(P/F,10\%,5)(A/P,10\%,10)
- (+) $10,000(A/F,10\%,10)
- (+) $50,000(A/F,10\%,10)
- (-) $5,000(A/F,10\%,10)
- =
6.4 Example Artificial Carpet (Surface)

• A = P(i) for a perpetual life project
• Annual Cost of Installation:
• =\$150,000 (.10) = \$15,000/ year
• Annual Maintenance = \$5,000/year

• Total: \$15,000 + \$5,000 = \$20,000/Yr
• Choose B, cost less per year!
Chapter 6: Summary

- Annual Worth is popular…
- Fairly simple to understand…
- Variant of the Present worth approach…
- Eliminates lowest common multiples of lives when alternatives possess unequal lives
- Understood by decision makers
- Consistent with present worth
Assignments and Announcements

- Assignments due at the beginning of next class:
  - Do the online quizzes for chapter 6.
  - Read 7.1, 7.2, 7.3