Chapter 5:
PRESENT WORTH ANALYSIS

Session 12-13-14
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PRESENT WORTH ANALYSIS

• So Far, Present worth computations have been made for one project or alternative.
• In chapter 5, techniques for comparing two or more mutually exclusive alternatives by the present worth method are treated.
• We will also cover, Future Worth analysis, capitalized cost, payback period, and bond analysis which all use present worth relations to analyze alternatives.
Terminology

- **Present Worth (PW)** is also called Discounted Cash Flow (DCF), **Present Value (PV)**, and **Net Present Value (NPV)**.
- The **interest rate** is also referred to as the **discount rate**.
Topics to Be Covered in Today’s Lecture

- Section 5.1: Formulating Mutually Exclusive Alternatives
- Section 5.2: PW Analysis of Equal-life Alternatives
Section 5.1: Mutually Exclusive Alternatives

- One of the important functions of financial management and engineering is the creation of “alternatives”.
- If there are no alternatives to consider then there really is no problem to solve!
- Given a set of “feasible” alternatives, engineering economy attempts to identify the “best” economic approach to a given problem.
- Part of Engineering Economy is the selection and execution of the best alternative from among a set of feasible alternatives.
5.1 Projects to Alternatives

• Creation of Alternatives.

Ideas, Data, Experience, Plans, And Estimates

Generation of Proposals

P1  P2  Pn
5.1 Projects to Alternatives

- Proposal Assessment.

Economic Analysis And Assessment

Viable

P_{OK} P_{OK} P_{j} P_{k}

Infeasible or Rejected!

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5.1 Assessing Alternatives

- Feasible Alternatives

**Economic Analysis And Assessment**

- Feasible Set
  - $P_{OK}$

Mutually Exclusive Set

- $P_{OK}$

OR

Independent Set
5.1. Three Types of Categories:

1. The Single Project

2. Mutually Exclusive Set,

3. Independent Project Set.
5.1 The Single Project

• Called “The Unconstrained Project Selection Problem;
• No comparison to competing or alternative projects;
• Acceptance or Rejection is based upon:
  – Comparison with the firm’s opportunity cost;
  – Opportunity Cost is always present;
5.1 The Single Project: Opportunity Cost

- A firm must always compare the single project to two alternatives;
  - Do Nothing (reject the project) or,
  - Accept the project

- Do Nothing:
  - Involve the alternative use of the firm’s funds that could be invested in the project!
  - The firm will always have the option to invest the owner’s funds in the BEST alternative use:
    - Invest at the firm’s MARR;
    - Market Place opportunities for external investments with similar risks as the proposed project;
5.1 The Single Project: The Implicit Risk – Single Project

- For the single project then:
  - There is an implicit comparison with the firm’s MARR vs. investing outside of the firm;
  - Simple: evaluate the single project’s present worth at the firm’s MARR;
  - If the present worth > 0 - - conditionally accept the project;
  - Else, reject the project!
5.1 Mutually Exclusive Set

1. Mutually Exclusive (ME) Set

- Only one of the feasible (viable) projects can be selected from the set.
- Once selected, the others in the set are “excluded”.
- Each of the identified feasible (viable) projects is (are) considered an “alternative”.

- It is assumed the set is comprised of “do-able”, feasible alternatives.
- ME alternatives compete with each other!
5.1 Alternatives

Problem

Do Nothing

Alt. 1

Alt. 2

Alt. m

Analysis

Selection

Execute!
5.1 Alternatives: The Selected Alternative

Selection is dependent upon the data, life, discount rate, and assumptions made.
5.1 The Independent Project Set

• Independent Set
  – Given the alternatives in the set:
  – More than one can be selected;
  – Deal with budget limitations;
  – Project Dependencies and relationships.

• More Involved Analysis –
  – Often formulated as a 0-1 Linear Programming model
  – With constraints and an objective function.

• May or may not compete with each other – depends upon the conditions and constraints that define the set!

• Independent Project analysis can become computationally involved!

• See Chapter 12 for a detailed analysis of the independent set problem!
5.1 The Do-Nothing Alternative

- Do-nothing (DN) can be a viable alternative;
- Maintain the status quo;
  - However, DN may have a substantial cost associated and may not be desirable.
- DN may not be an option;
  - It might be that something has to be done and maintaining the status quo is NOT an option!

- Rejection – Default to DN!
  - It could occur that an analysis reveals that all of the new alternatives are not economically desirable then:
  - Could default to the DN option by the rejection of the other alternatives!
  - Always study and understand the current, in-place system!
5.2 THE PRESENT WORTH METHOD

A process of obtaining the equivalent worth of future cash flows BACK to some point in time.

– called the Present Worth Method.

At an interest rate usually equal to or greater than the Organization’s established MARR.
5.2 Present Worth – A Function of the assumed interest rate.

• If the cash flow contains a mixture of positive and negative cash flows –

• We calculate:
  – $PW(+ \text{ Cash Flows})$ at $i\%$;
  – $PW(\ "-\" \text{ Cash Flows})$ at $i\%$;
  – Add the result!

• We can add the two results since the equivalent cash flows occur at the same point in time!
5.2 Present Worth – A Function of the assumed interest rate.

- If $P(i\%) > 0$ then the project is deemed acceptable.
- If $P(i\%) < 0$ then the project is deemed unacceptable!
- If the net present worth $= 0$ then,
  - The project earns exactly $i\%$ return
  - Indifferent unless we choose to accept the project at $i\%$. 

5.2 Present Worth – A Function of the assumed interest rate.

- Present Worth transforms the future cash flows into:
  - Equivalent Dollars NOW!
  - One then COMPARES alternatives using the present dollars of each alternative.

- Problem:
  - Present Worth requires that the lives of all alternatives be EQUAL.
5.2 Equal Lives

- Present Worth is a method that assumes the project life or time span of all alternatives are EQUAL.
- Assume two projects, A and B.
  - Assume:
    - \( n_A = 5 \) years;
    - \( n_B = 7 \) Years;
  - You can compute \( P_A \) and \( P_B \) however;
    - The two amounts **cannot be compared** at \( t = 0 \)
    - Because of unequal lives!
5.2 All Cost Alternatives

- If the alternatives involves “future costs” and the lives are equal then:
  - Compute the PW(i%) of all alternatives;
  - Select the alternative with the LOWEST present worth cost.
5.2 All Revenue Alternatives

- If the alternatives involve all revenues (+) cash flows and the lives are equal;
  - Compute the present worth of all alternatives and the same interest rate, i% and;
  - Select the alternative with the greatest present worth at i%.
5.2 Mixture of Costs and Revenues

• Assuming the lives of all alternatives are equal then;
  – Compute the present worth at i% of all alternatives;
  – Select the alternative with the greatest present worth at i%.
5.2 IF Project Set is Independent…

• Assuming an independent set then:
  – Assuming the lives of the alternatives are equal;
  – Calculate the present worth of ALL alternatives in the set;
  – Then the accepted set will be all projects with a positive present worth at i%.
  – Then, a budget constraint – if any – must be applied.
5.2 Points to Remember:

• Project lives of all alternatives must be equal or adjusted to be equal.

• Given the discount rate – i%
  – The same interest rate is applied to projects in the set.

• The interest rate must be at the firm’s MARR or can be higher but not lower!
5.2 Example: Three Alternatives

- Assume $i = 10\%$ per year

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<tbody>
<tr>
<td>A1</td>
<td>Electric Power</td>
<td>-2500</td>
<td>-900</td>
<td>+200</td>
<td>5 years</td>
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<tr>
<td>A2</td>
<td>Gas Power</td>
<td>-3500</td>
<td>-700</td>
<td>+350</td>
<td>5 years</td>
</tr>
<tr>
<td>A3</td>
<td>Solar Power</td>
<td>-6000</td>
<td>-50</td>
<td>+100</td>
<td>5 years</td>
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Which Alternative – if any, Should be selected based upon a present worth worth analysis?
5.2 Example: Cash Flow Diagrams

A₁: Electric

A = -900/Yr.

F_{SV} = 200

A₂: Gas

A = -700/Yr.

F_{SV} = 350

A₃: Solar

A = -50/Yr.

F_{SV} = 100

\( i = 10\%/yr \) and \( n = 5 \)
5.2 Calculate the Present Worth's

- Present Worth's are:

1. \( PW_{\text{Elec.}} = -2500 - 900(P/A,10\%,5) + 200(P/F,10\%,5) = \$-5788 \)

2. \( PW_{\text{Gas}} = -3500 - 700(P/A,10\%,5) + 350(P/F,10\%,5) = \$-5936 \)

3. \( PW_{\text{Solar}} = -6000 - 50(P/A,10\%,5) + 100(P/F,10\%,5) = \$-6127 \)

Select “Electric” which has the min. PW Cost!
Assignments and Announcements

- Assignments due at the beginning of next class:
  - Read sections 5.3, 5.4, 5.5
Topics to Be Covered in Today’s Lecture

- Section 5.3: PW Analysis of Different-life Alternatives;
- Section 5.4: Future Worth Analysis
- Section 5.5: Capitalized Cost Calculation and Analysis
Section 5.3 Different Lives

- With alternatives with Un-equal lives the rule is:
  - The PW of the alternatives must be compared over the same number of years.

- Called “The Equal Service” requirement
5.3 Two Approaches for Unequal Lives

• IF present worth is to be applied, there are two approaches one can take to the unequal life situation:

1. **Least common multiple (LCM) of their lives:** Compare the alternatives over a period of time equal to the least common multiple (LCM) of their lives.

2. **The planning horizon approach:** Compare the alternatives using a study period of length n years, which does not necessarily take into consideration the useful lives of the alternatives!
5.3 LCM Approach

• The **assumptions** of a PW analysis of different-life alternatives are as follows:
  1. The service provided by the alternatives will be needed for the LCM of years or more.
  2. The selected alternative will be repeated over each life cycle of the LCM in **exactly the same manner**.
  3. The **cash flow estimates will be the same** in every life cycle.
## 5.3 PW – LCM Example

- Two Location Alternatives, A and B where one can lease one of two locations.

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<tr>
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<th>Location A</th>
<th>Location B</th>
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<tbody>
<tr>
<td>First cost, $</td>
<td>-15,000</td>
<td>-18,000</td>
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<tr>
<td>Annual lease cost, $</td>
<td>-3,500</td>
<td>-3,100</td>
</tr>
<tr>
<td>Deposit return, $</td>
<td>1,000</td>
<td>2,000</td>
</tr>
<tr>
<td>Lease term, years</td>
<td>6</td>
<td>9</td>
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Note: The lives are unequal. The LCM of 6 and 9 = 18 years!
5.3 LCM Example: Required to Find:

- Which option is preferred if the interest rate is 15%/year?

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For now, assume there is no study life constraint – so apply the LCM approach.
5.3 Unequal Lives: 2 Alternatives

A

Cycle 1 for A | Cycle 2 for A | Cycle 3 for A

6 years | 6 years | 6 years

B

Cycle 1 for B | Cycle 2 for B

9 years | 9 years

18 years

\( i = 15\% \) per year

\[ \text{LCM}(6,9) = 18 \text{ year study period will apply for present worth} \]
5.3 LCM Example where “n” = 18 yrs.

• The Cash Flow Diagrams are:
5.3 LCM Example Present Worth's

• Since the leases have different terms (lives), compare them over the LCM of 18 years.
• For life cycles after the first, the first cost is repeated in year 0 of the new cycle, which is the last year of the previous cycle.
• These are years 6 and 12 for location A and year 9 for B.
• Calculate PW at 15% over 18 years.
5.3 PW Calculation for A and B - 18 yrs

- \( PW_A = -15,000 \times \frac{1}{(1 + 0.15)^6} + 1000 \times \frac{1}{(1 + 0.15)^6} - 15,000 \times \frac{1}{(1 + 0.15)^{12}} + 1000 \times \frac{1}{(1 + 0.15)^{12}} + 1000 \times \frac{1}{(1 + 0.15)^{18}} - 3500 \times \frac{1}{(1 + 0.15)^{18}} = -45,036 \)

- \( PW_B = -18,000 \times \frac{1}{(1 + 0.15)^9} + 2000 \times \frac{1}{(1 + 0.15)^9} + 2000 \times \frac{1}{(1 + 0.15)^{18}} - 3100 \times \frac{1}{(1 + 0.15)^{18}} = -41,384 \)

Select “B”: Lowest PW Cost @ 15%
5.3 LCM Observations

- For the LCM method:
  - Becomes tedious;
  - Numerous calculations to perform;
  - The assumptions of repeatability of future cost/revenue patterns may be unrealistic.

- However, in the absence of additional information regarding future cash flows, this is an acceptable analysis approach for the PW method.
5.3 The *Study Period* Approach

An alternative method;

- Impress a *study period* (SP) on all of the alternatives;
- A time horizon is selected in advance;
- Only the cash flows occurring within that time span are considered relevant;
- May require assumptions concerning some of the cash flows.
- Common approach and simplifies the analysis somewhat.
5.3 Example Problem with a 5-yr SP

- Assume a 5- year Study Period for both options:

For a 5-year study period no cycle repeats are necessary.

\[ PW_A = -15,000 - 3500(P/A,15\%,5) + 1000(P/F,15\%,5) \]
\[ = -$26,236 \]

\[ PW_B = -18,000- 3100(P/A,15\%,5) + 2000(P/F,15\%,5) \]
\[ = -$27,397 \]

Location A is now the better choice.

Note: The assumptions made for the A and B alternatives! Do not expect the same result with a study period approach vs. the LCM approach!
Section 5.4 Future Worth Analysis

• In some applications, management may prefer a future worth analysis;

• Analysis is straight forward:

• Find $P_0$ of each alternative:

• Then compute $F_n$ at the same interest rate used to find $P_0$ of each alternative.

• For a study period approach, use the appropriate value of “n” to take forward.
5.4 Future Worth Approach (FW)

- Especially applicable to large capital investment decisions.
- Applications for the FW approach:
  - **Future wealth maximization**;
  - **Projects that do not come on line until the end of the investment (construction) period**:
    - Power Generation Facilities
    - Toll Roads
    - Large building projects
    - Etc.
  - Also useful is the asset might be sold before the expected life is reached.
Section 5.5: Capitalized Cost Calculation and Analysis

- CAPITALIZED COST- the present worth of a project which lasts forever.
- Government Projects;
- Roads, Dams, Bridges, project that possess perpetual life;
- Permanent and charitable organization endowments.
- Infinite analysis period;
- “n” in the problem is either very long, indefinite, or infinity.
5.5 Derivation of Capitalized Cost

• We start with the relationship:
  – \( P = A[P/A,i\%,n] \)
  – Next, what happens to the P/A factor when we let \( n \) approach infinity.
  – Some “math” follows.
5.5 P/A where “n” goes to infinity

• The P/A factor is:

\[ P = A \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right] \]

• On the right hand side, divide both numerator and denominator by \((1+i)^n\)

\[ P = A \left[ \frac{1 - \frac{1}{(1+i)^n}}{i} \right] \]
5.5 CC Derivation…

• Repeating:

\[ P = A \left[ 1 - \frac{1}{(1+i)^n} \right] \]

• If “n” approaches \( \infty \) the above reduces to:

\[ P = \frac{A}{i} \]
5.5 CC Explained

- For this class of problems, we can use the term “CC” in place of P.
- Restate:

\[ CC = \frac{A}{i} \]

- Or,

\[ CC = \frac{AW}{i} \]
5.5 CC –type problems.

• CC-type problems vary from very simple to somewhat complex.
• Consider a “simple” CC-type problem.
• Assume that $10,000 can earn 20% per year;
• How much money can be withdrawn forever from this account?
5.5 CC Example

- Draw a Cash Flow Diagram

\[ P = \frac{A}{i} \rightarrow A = P(i) \]

\[ A = \$10,000(0.20) = \$2,000 \text{ per period} \]
5.5 CC – Recurring and Non-recurring

- More complex problems will have two types of costs associated;
  - Recurring and,
  - Non-recurring.
- Recurring – Periodic and repeat.
- Non-recurring – One time present or future cash flows
- For more complex CC problems one must separate the recurring from the non-recurring.
5.5 CC – A More Involved Example
(Example 5.4)

• Problem Description

• The property appraisal district for a local county has just installed new software to track residential market values for property tax computations.

• The manager wants to know the total equivalent cost of all future costs incurred when the three county judges agreed to purchase the software system.

• If the new system will be used for the indefinite future, find the equivalent value (a) now and (b) for each year hereafter.
5.5 CC Example - continued

• Problem Parameters:

The system has an installed cost of $150,000 and an additional cost of $50,000 after 10 years.

The annual software maintenance contract cost is $5000 for the first 4 years and $8000 thereafter.

In addition, there is expected to be a recurring major upgrade cost of $15,000 every 13 years. Assume that \( i = 5\% \) per year for county funds.
5.5 CC Example - continued

• Required to aid the manager in determining:

1. The present worth equivalent cost at 5%;
2. The future annual amount ($/year) that the county will be committed.
5.5 CC – The Steps To Follow

- 1. DRAW a cash flow diagram!

This Step is critical to the subsequent analysis!
5. 5 CC – Second Step

- Find the present worth of all nonrecurring costs
- Could call this amount $CC_1$
- Nonrecurring costs are:
  - $150,000$ time $t = 0$ investment;
  - $50,000$ at time $t = 10$.
  - “$i$” = 5% per year.

$$CC_1 = -150,000 - 50,000(P/F,5\%,10) = -$180,695$$
5.5 CC – Third Step

• CONVERT the recurring costs into an annualized equivalent amount.

• Could call this “A₁”
  – For the problem, we have $15,000 every 13 years;
  – Using the (A/F) factor we have:

\[
A₁ = -15,000(A/F,5\%,13) = $-847.00
\]

Important: This same value applies to all other 13-year time periods as well!
5.5 The Rest of the Problem

• The other issues involve:

The capitalized cost for the two annual maintenance cost series may be determined in either of two ways:

(1) consider a series of $-5000 from now to infinity and find the present worth of -$8000 - ($-5000) = $-3000 from year 5 on; or...
5. 5 The Rest of the Problem

• The other issues involve:

(2) find the CC of $-5000 for 4 years and the present worth of $-8000 from year 5 to infinity. Using the first method, the annual cost \( (A_2) \) is $-5000 forever.

The capitalized cost \( CC_2 \) of $-3000 from year 5 to infinity is found using:

\[
CC_2 = \frac{-3000}{0.05} (P / F, 5\%, 4) = $ -49,362
\]
5. 5 The Rest of the Problem

• The other issues involve:

The two annual cost series are converted into a capitalized cost $CC_3$

$$CC_3 = \frac{(A1+A2)}{i}$$

$$= \frac{(-847+(-5000))}{0.05} = -$116,940$$
5.5 Total $\text{CC}_T$:

- The total $\text{CC}_T$ is found by:

- $\text{CC}_T = -180,695 - 49,362 - 116,940$

- $\text{CC}_T = -$346,997

- The equivalent $A$ per year forever is

- $A = P(i) = -$346,997(0.05) = $17,350/yr
5.5 CC Interpretation for This Problem

- One major point to consider!

- The $-346,997 represents the one-time t = 0 amount that if invested at 5%/year would fund the future cash flows as shown on the cash flow diagram from now to infinity!

- CC-type problems can be involved and take careful thought in setting up the correct relationships
5.5 CC Problem: Public Works Example (Example 5.5)

• Good Example of a Public Sector type problem:

Two sites are currently under consideration for a bridge to cross a river in New York:

• The north site, which connects a major state highway with an interstate loop around the city, would alleviate much of the local through traffic. The disadvantages of this site are that the bridge would do little to ease local traffic congestion during rush hours, and the bridge would have to stretch from one hill to another to span the widest part of the river, railroad tracks, and local highways below. This bridge would therefore be a suspension bridge.

• The south site would require a much shorter span, allowing for construction of a truss bridge, but it would require new road construction.
5.5 CC Problem: Public Works Example

• Problem Parameters

The suspension bridge will cost $50 million with annual inspection and maintenance - costs of $35,000. In addition, the concrete deck would have to be resurfaced every 10 years at a cost of $100,000. The truss bridge and approach roads are expected to cost $25 million and have annual maintenance costs of $20,000.
5.5 CC Problem: Public Works Example

- **Problem Parameters**

  - The bridge would have to be painted every 3 years at a cost of $40,000.
  - In addition, the bridge would have to be sandblasted every 10 years at a cost of $190,000.
  - The cost of purchasing right-of-way is expected to be $2 million for the suspension bridge and $15 million for the truss bridge.
  - Compare the alternatives on the basis of their capitalized cost if the interest rate is 6% per year.

**Two, Mutually Exclusive Alternatives: Select the best alternative based upon a CC analysis**
5.5 Bridge Alternatives: Suspension

- Cash Flow Diagrams

Suspension Bridge Alternative

$50 Million

$2 Million

$i = 6\%$/year

$35,000/yr$

$100,000$
5.5 Suspension Bridge Analysis

• \( CC_1 = -52 \text{ million at } t = 0. \)

\[
A_1 = -$35,000 \\
A_2 = -100,000(A/F, 6\%, 10) = -$7,587 \\
CC_2 = \frac{A_1 + A_2}{i} = \frac{-35,000 + (-7,587)}{0.06} = -$709,783.
\]

**Total CC – suspension bridge is:**

-52 million + (709,783) = **-$52.71 million**
5. 5 Truss Bridge Alternative

- For the Truss Bridge Alternative: Cash Flow Diagram:

Truss Design:

-25M +(-15M)

0 1 2 3 4 5 6 7 8 9 10 11 ..... 

A. Maint. = $20,000/yr

Paint: -40,000 Paint: -40,000 Paint: -40,000

Sandblast: -190,000

\[ i = 6\%/\text{year} \]
5.5 Truss Bridge Alternative

1. **CC₁ Initial Cost:**
   
   $-25\text{M} + (-15\text{M}) = -40\text{M}

**Truss Design:**

\[
\begin{array}{cccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & \ldots \ \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\
-25\text{M} + (-15\text{M}) & A. Maint. = $20,000/yr & \text{Paint: -40,000} & \text{Paint: -40,000} & \text{Paint: -40,000} & \text{Sandblast: -190,000} & \\
\end{array}
\]

\[i = 6\%/year\]
5. 5 Truss Bridge Alternative

2. Annual Maintenance is already an “A” amount so: $A_1 = -$20,000/year

Truss Design:

A. Maint. = $20,000/yr

$-25M +(-15M)$

Paint: $-40,000$

Paint: $-40,000$

Paint: $-40,000$

Sandblast: $-190,000$

$i = 6\%/year$

$n = \infty$
5. 5 Truss Bridge Alternative

3. $A_2$: Annual Cost of Painting

Truss Design:

\[
i = 6\%/\text{year}
\]

\[
\begin{array}{cccccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & \ldots
\end{array}
\]

-25M +(-2M)

-25M +(-2M)

Use A/F,6%,3

A. Maint. = $20,000/yr

Paint: -$40,000 Paint: -$40,000 Paint: -$40,000

Sandblast: -190,000

For any given cycle of painting compute:

\[
A_2 = -$40,000(A/F,6\%,3) = -$12,564/\text{year}
\]
5. 5 Truss Bridge Alternative

3. A₃ Annual Cost of Sandblasting

Truss Design: $i = 6\%$/year

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<td>A. Maint. = $20,000/yr</td>
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<tr>
<td>Use The A/F,6%,10 to convert to an equivalent $/year amount</td>
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<td>-25M +(-2M)</td>
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<tr>
<td>Paint: -40,000</td>
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<td>Paint: -40,000</td>
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<td>Paint: -40,000</td>
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<tr>
<td>Sandblast: -190,000</td>
<td></td>
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</tr>
</tbody>
</table>

For any given cycle of Sandblasting Compute

$$A₃ = -$190,000(A/F,6\%,10) = -$14,421$$
5.5 Bridge Summary for CC(6%)

- \[ CC_2 = (A_1 + A_2 + A_3)/i \]
- \[ CC_2 = -(20,000 + 12,564 + 14,421)/0.06 \]
- \[ CC_2 = -$783,083/\text{year} \]
- \[ CC_{\text{Total}} = CC_1 + CC_2 = -40.783 \text{ million} \]

- \[ CC_{\text{Suspension}} = -$52.71 \text{ million} \]
- \[ CCT_{\text{Russ}} = -40.783 \text{ million} \]
- Select the Truss Design!
Assignments and Announcements

- Assignments due at the beginning of next class:
  - Finish Reading chapter 5 (5.6 and 5.8 only)
Topics to Be Covered in Today’s Lecture

- Section 5.6: Payback Period Analysis
- Section 5.8: Present Worth of Bonds
Section 5.6 Payback Period Analysis

- Payback Analysis – PB: Extension of Present Worth
- Estimate of the time, \( n_p \) (payback period) to recover the initial investment in a project. Generally not an integer.
- A rate of return should be stated.
- Two forms:
  1. With 0% interest;
  2. With an assumed interest rate (also called discounted payback analysis)
5.6 Payback Period Analysis-RULE

• Never use PB as the primary means of making an accept-reject decision on an alternative!
• Often used as a screening technique or preliminary analysis tool.
• Historically, this method was a primary analysis tool and often resulted in incorrect selections.
• To apply, the cash flows must have at least one (+) cash flow in the sequence.
5.6 Payback - Formulation

• The formal relationship defining PB is:

\[ 0 = -P + \sum_{t=1}^{t=n_p} NCF_t (P / F, i\%, t). \]

• P is the initial investment or first cost
• \( NCF_t \) is the estimated Net Cash Flow for each year \( t = \) Inflows- outflows
• This is for the GENERAL case!
5.6 PB – 0% Interest Rate

• If the interest rate is “0”% we have:

\[-P + \sum_{t=1}^{t=n_p} NCF_t.\]

Which is the algebraic sum of all cash flows!
5.6 Special Case for PB:

- If the future cash flows represent a uniform series the PB analysis is:

\[ 0 = -P + NCF \left( P / A, i\%, n_p \right) \]
5.6 Example 1 for Payback

Consider a 5-year cash flow as shown.

<table>
<thead>
<tr>
<th>E.O.Y.</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-$30,000.00</td>
</tr>
<tr>
<td>1</td>
<td>-$4,000.00</td>
</tr>
<tr>
<td>2</td>
<td>$15,000.00</td>
</tr>
<tr>
<td>3</td>
<td>$16,000.00</td>
</tr>
<tr>
<td>4</td>
<td>$8,000.00</td>
</tr>
<tr>
<td>5</td>
<td>$8,000.00</td>
</tr>
<tr>
<td></td>
<td>$13,000.00</td>
</tr>
</tbody>
</table>

At a 0% interest rate, how long does it take to recover (pay back) this investment?
### 5.6 Payback Example: 0% Interest

- Form the Cumulative Cash Flow Amounts.

<table>
<thead>
<tr>
<th>E.O.Y.</th>
<th>Cash Flow</th>
<th>C.C. Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-$30,000.00</td>
<td>-$30,000.00</td>
</tr>
<tr>
<td>1</td>
<td>-$4,000.00</td>
<td>-$34,000.00</td>
</tr>
<tr>
<td>2</td>
<td>$15,000.00</td>
<td>-$19,000.00</td>
</tr>
<tr>
<td>3</td>
<td>$16,000.00</td>
<td>-$3,000.00</td>
</tr>
<tr>
<td>4</td>
<td>$8,000.00</td>
<td>$5,000.00</td>
</tr>
<tr>
<td>5</td>
<td>$8,000.00</td>
<td>$13,000.00</td>
</tr>
</tbody>
</table>

Compute the cumulative Cash Flow amounts as:

$13,000.00

PW(0%)
5.6 Payback Example: 0% Interest

- Form the Cumulative Cash Flow Amounts.

<table>
<thead>
<tr>
<th>E.O.Y.</th>
<th>Cash Flow</th>
<th>C.C. Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-$30,000.00</td>
<td>-$30,000.00</td>
</tr>
<tr>
<td>1</td>
<td>-$4,000.00</td>
<td>-$34,000.00</td>
</tr>
<tr>
<td>2</td>
<td>$15,000.00</td>
<td>-$19,000.00</td>
</tr>
<tr>
<td>3</td>
<td>$16,000.00</td>
<td>-$3,000.00</td>
</tr>
<tr>
<td>4</td>
<td>$8,000.00</td>
<td>$5,000.00</td>
</tr>
<tr>
<td>5</td>
<td>$8,000.00</td>
<td>$13,000.00</td>
</tr>
<tr>
<td></td>
<td>$13,000.00</td>
<td></td>
</tr>
</tbody>
</table>

Note: The cumulative cash flow Amounts go from (-) to (+) Between years 3 and 4.
5.6 Example: $3 < n_p < 4$

- Payback is between 3 and 4 years.
- Get “fancy” and interpolate as:

<table>
<thead>
<tr>
<th></th>
<th>Cumulative Cash Flow Amts.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$16,000.00</td>
</tr>
<tr>
<td>4</td>
<td>$8,000.00</td>
</tr>
</tbody>
</table>

\[-3,000\]
\[0\]
\[0.375 \text{ (yrs)}\]
\[+5,000\]

Payback Period = 3.375 years (really 4 years!)
5.6 Same Problem: Set $i = 10\%$

- Perform Discounted Payback at $i = 10\%$.
- Cannot simply use the cumulative cash flow.
- Have to form the discounted cash flow cumulative amount and look for the first sign change from (-) to (+).
- Example calculations follow.
### 5.6 %-Year Example at 10%

- Discounted Payback at 10%: tabulations:

<table>
<thead>
<tr>
<th>E.O.Y.</th>
<th>Cash Flow ((1))</th>
<th>(P/F,10%, t) ((2))</th>
<th>Dis. Incmmt ((3)=(1)(2))</th>
<th>Accum Disc. Amts ((4)=\text{Cuml. Sum of (3)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-$30,000.00</td>
<td>1.00000</td>
<td>-$30,000.00</td>
<td>-$30,000.00</td>
</tr>
<tr>
<td>1</td>
<td>-$4,000.00</td>
<td>0.90909</td>
<td>-$3,636.36</td>
<td>-$33,636.36</td>
</tr>
<tr>
<td>2</td>
<td>$15,000.00</td>
<td>0.82645</td>
<td>$12,396.69</td>
<td>-$21,239.67</td>
</tr>
<tr>
<td>3</td>
<td>$16,000.00</td>
<td>0.75131</td>
<td>$12,021.04</td>
<td>-$9,218.63</td>
</tr>
<tr>
<td>4</td>
<td>$8,000.00</td>
<td>0.68301</td>
<td>$5,464.11</td>
<td>-$3,754.52</td>
</tr>
<tr>
<td>5</td>
<td>$8,000.00</td>
<td>0.62092</td>
<td>$4,967.37</td>
<td>$1,212.85</td>
</tr>
</tbody>
</table>

Locate the time periods where the first sign change from (-) to (+) occurs!
### 5.6 %-Year Example at 10%

- PB is between 4 and 5 years at 10%

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-$30,000.00</td>
<td>1.00000</td>
<td>-$30,000.00</td>
<td>-$30,000.00</td>
</tr>
<tr>
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<td>0.62092</td>
<td>$4,967.37</td>
<td>$1,212.85</td>
</tr>
</tbody>
</table>

Locate the time periods where the first sign change From (-) to (+) occurs!
5.6 Payback Period at 10%

- Interpolation of PB time at 5%.

<table>
<thead>
<tr>
<th>CF(t)</th>
<th>Accum. DC. CF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8,000.00</td>
<td>-$3,754.52</td>
</tr>
<tr>
<td>$8,000.00</td>
<td>$1,212.85</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
4 & \quad -3755 \\
0 & \quad np \approx \frac{3755}{4968} = 0.76 \text{ (years)} \\
5 & \quad +1213
\end{align*}
\]

At 10%, the Payback Time Period approx. **4.76 yrs**
5.6 Comparing Pay Back Periods

- At 0% the Payback was 3.375 yrs.
- At 10%, the Payback was 4.76 yrs.
- Generalize:
  - At higher and higher discounts rates the payback period for the same cash flow will increase as the applied interest rate increases.
5.6 Payback - Interpretations

• A managerial philosophy is: a shorter payback period is preferred to a longer payback period.

• Not a preferred method for final decision making – rather, use as a screening tool.

• Ignores all cash flows after the payback time period – next example.

• May not use all of the cash flows in the cash flow sequence.
5.6 Cash Flows Ignored after PB time.

Consider:

<table>
<thead>
<tr>
<th>E.O.Y.</th>
<th>Cash Flow (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-$10,000.00</td>
</tr>
<tr>
<td>1</td>
<td>$2,000.00</td>
</tr>
<tr>
<td>2</td>
<td>$6,000.00</td>
</tr>
<tr>
<td>3</td>
<td>$8,000.00</td>
</tr>
<tr>
<td>4</td>
<td>$4,000.00</td>
</tr>
<tr>
<td>5</td>
<td>$1,000.00</td>
</tr>
</tbody>
</table>
### 5.6 Tabular Analysis for PB

<table>
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<th></th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>-$10,000.00</td>
<td>-$10,000.00</td>
<td>1.00000</td>
<td>-$10,000.00</td>
<td>-$10,000.00</td>
</tr>
<tr>
<td>1</td>
<td>$2,000.00</td>
<td>-$8,000.00</td>
<td>0.92593</td>
<td>$1,851.85</td>
<td>-$8,148.15</td>
</tr>
<tr>
<td>2</td>
<td>$6,000.00</td>
<td>-$2,000.00</td>
<td>0.85734</td>
<td>$5,144.03</td>
<td>-$3,004.12</td>
</tr>
<tr>
<td>3</td>
<td>$8,000.00</td>
<td>$6,000.00</td>
<td>0.79383</td>
<td>$6,350.66</td>
<td>$3,346.54</td>
</tr>
<tr>
<td>4</td>
<td>$4,000.00</td>
<td>$10,000.00</td>
<td>0.73503</td>
<td>$2,940.12</td>
<td>$6,286.66</td>
</tr>
<tr>
<td>5</td>
<td>$1,000.00</td>
<td>$11,000.00</td>
<td>0.68058</td>
<td>$680.58</td>
<td>$6,967.25</td>
</tr>
</tbody>
</table>

- PB is between 2 and 3 years (2.45 years) – say 3 years.
- The years 4 and 5 cash flows are NOT used in the calculations.
- None of the cash flows AFTER the payback time are involved in the analysis – both (+) and (-)!
Section 5.8: Present Worth of Bonds

- Bond problems – typical present worth application;
- Common analysis problems in the world of finance;
- Bond – basically an “iou”;
- Bond – represent “debt” financing;
- Firm’s have bonds sold for them to raise capital;
- Bonds pay a stated rate of interest to the bond holder for a specified period of time.
5.8 Bonds as Financing Instruments

- Bonds are a method of raising debt capital to assist in financing operations.

The Firm → Investment Banks → Sell Bonds in Mkt. Place → $1,000

The public then “bid” on the bonds in the Bond market which establishes The price per bond.

The bonds are then sold in the market with commissions going to the investment bankers and the proceeds to the firm.
5.8 Bonds: Parameters

• The important parameters of a bond:
  1. Face Value ($100, $1000, $5000,..);
  2. Life of the bonds (years);
  3. Nominal interest rate/interest payment period; (coupon rate).
5.8 Types of Bonds Treasure Bonds

- Treasury Bonds
  - Federal Government;
  - Backed by the Federal government;
  - Life
    - Bills: Less than one year;
    - Notes: 2-10 years;
    - Bonds: 10 – 30 years
5.8 Municipal Bonds

- Issued by Local Governments;
- Bond interest may be tax exempt;
- Bond holders paid back from tax revenues;
- Pay fairly low interest rates.
5.8 Mortgage Bonds

- Issued by Corporations;
- May be secured by the assets of the issuing corporation;
- The corporation can be foreclosed on if not paid back.
5.8 Debenture Bonds

- Issued by Corporations;
- Not backed by assets of the firm;
- Backed mostly on the “strength” of the corporation;
- Generally pay a higher rate of interest because of “risk”
- May be convertible into stock.
5.8 Cash Flow Profile of a bond

• Typical bond cash flow to the bond buyer:
5.8 Bond Interest

- A bond represents a contract between the issuing firm and the current bond holder.
- Bonds will have a stated rate of interest and timing of the interest payments to the current bondholder.
- The current bond holder receives the periodic interest as long as he/she hold the bonds.

\[ I = \frac{(\text{face value})(\text{bond coupon rate})}{\text{number of payment periods per year}} \]

\[ = \frac{Vb}{c} \]
5.8 Bond Interest Rates

- Typical interest rate statement:
- “$5,000, 10-year bond with interest paid 6% per year paid quarterly.”
- Face value = $5,000;
- Life will be 10 years;
- Interest will be paid quarterly;
- Rate per quarter: 0.06/4 = 1.5%/qtr.
- There will be (10)(4) = 40 future interest payments.
5.8 Bond Example

• Since the Face Value = $5,000, this amount will be paid to the current bond holder at the end of 10 years or 40 quarters.

• Interest amount per quarter:
  – \( I = V \text{(rate)} / \text{no. of pmt. Periods/year}; \)
  – \( I = $5,000(0.06/4) = $75.00 \text{ per quarter}. \)
5.8 Discounting a Bond

- Bonds are seldom purchased for their face values;
- Most of the time a bond is “discounted” in the bond market;
- But the face value and the amount of the periodic interest rates remain unchanged.
- Bond prices are “bid” in the bond market which impacts their selling price.
5.8 Bond Problem Example

- A $5,000, 4.5% paid semi-annually bond with a 10 year life is under consideration for purchase.
- As the potential buyer you require an 8% c.q. rate of return on your “investment”.
- What would you be willing to pay for this bond now in order to receive at least a 8% c.q. rate of return?
5.8 Analysis of the Bond Purchase

- Draw the cash flow diagram using “6 month periods” as the unit of time.
- Bond Interest = $5000(0.045)/2 = $112.50 every 6 months.

“n” for this problem is 20 since interest payments are semi-annual.
5.8 The Approach

- The potential bond buyer has the following future cash flows from this opportunity:

\[
\begin{array}{c|c|c|c|c|c}
0 & 1 & 2 & 3 & 4 & \hline \\
\$5,000 & & & & & \\
A = \$112.50 & & & & & \\
\end{array}
\]

- The purchaser requires 8% c.q. on this cash flow;

- So, discount (PW) at the investor’s required interest rate of 8% c.q.
5.8 The Approach - PW

- Investor requires 8% c.q.
- \( i/\text{qtr} = 0.8/4 = 2\% \) per quarter
- Eff quarterly rate is:
  - \( (1.02)^2 - 1 = 4.04\% \) per 6 months

Find the PW(4.04\%) of this cash flow!

PW = the max amount to pay for the bond!
5.8 PW Calculation

- \[ P = 112.50(P/A,4.04\%,20) + 5000(P/F,4.04\%,20). \]
- \[ P = $3788 \]
- Thus, if the bond can be purchased for 3788 or less, the investor will make his/her required rate of return (8%, c.q.).
- If the bond cost more than $3788, the investment is not worth it to the buyer.
5.8 Present Worth and Bonds

- The bond problem represents a common application of the present worth approach.
- Bond problems often require application of nominal and effective interest rates combined with PW analysis over a known life.
5.8 Present Worth and Bonds

- The key is to always discount the bond cash flow at the investor’s required effective interest rate.
- The interest payments are always calculated from the terms of the bond (coupon rate, frequency of payments, and bond life).
Chapter 5 Summary

• PW requires converting all cash flows to present dollars at a required MARR.

• Equal lives can be compared directly.

• Unequal lives – must apply:
  – LCM of lives or,
  – Impressed study period where some cash flows may have to be dropped in the longer lived alternative.
  – Must always use equal life patterns when applying Present Worth to alternatives.
Summary cont.

- Capitalized Cost is a present worth analysis approach for alternatives with infinite life.
- Payback analysis – estimate the number of years to pay back an original investment:
  - Estimates the time required to recover an initial investment;
  - At a 0% interest rate or,
  - At a specified rate;
  - NOT a method to apply for a correct analysis and does not support wealth maximization;
  - Used more for a preliminary or screening analysis.
Summary cont.

• Bonds
  – PW provides an analysis technique to evaluate bond purchases and bond yields.
  – PW is a common analysis tool for bond problems.
Summary cont.

- Remember the following:
  - Given a cash flow and an interest rate;
  - The present worth is a function of that interest rate;
  - If you re-evaluate the same cash flow at a different interest rate – the present value will change.
  - So, acceptance or rejection of a given cash flow can depend upon the interest rate used to determine the PV.
Summary cont.

• The present worth method constitutes the base method from which all of the other analysis approaches are generated.

• Present worth is always a proper approach if the lives of the alternatives are equal or assumed equal.

• Must have a discount rate BEFORE the analysis is conducted!
Assignments and Announcements

- Assignments due at the beginning of next class:
  - Finish Reading chapter 5
  - Online quizzes due before next class